Name: ------------------ **Quiz 1** **Time: 20 minutes**

ME:5160 **Fall 2024**

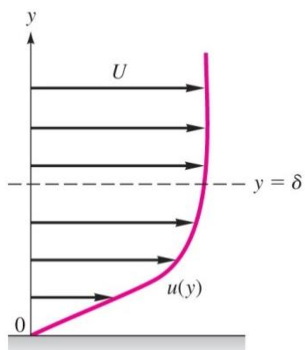
The exam is closed book and closed notes.

An approximation for the boundary-layer shape is the formula:

$$u(y) \approx U \sin\left(\frac{\pi y}{2\delta}\right), \quad 0 \le y \le \delta$$

Where U is the stream velocity far from the wall and δ is the boundary layer thickness, as in the Figure below. If the fluid is helium at 20°C and 1 atm (ρ = 0.1664 kg/m^3; μ = 1.97E-5 kg/m-s), and if U = 7.9 m/s and δ = 4.5 cm, use the formula to (*a*) estimate the wall shear stress τ_w in Pa, and (*b*) find the position in the boundary layer where τ is one-half of τ_w.

Hint: $\tau = \mu \frac{du}{dy}$



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Solution:

(a)

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \mu\left(U\frac{\pi}{2\delta}\cos\frac{\pi y}{2\delta}\right)_{y=0} = \frac{\pi\,\mu U}{2\,\delta} \qquad +4$$

$$\text{Numerical values}: \tau_w = \frac{\pi(1.97E{-}5\frac{kg}{m-s})(7.9\frac{m}{s})}{2(0.045m)} = \mathbf{5.43E\text{-}3\ Pa} \qquad +3$$

(b)

The variation of shear stress across the boundary layer is a cosine wave, $\tau = \mu\,(du/dy)$:

$$\tau(y) = \frac{\pi\mu U}{2\delta}\cos\left(\frac{\pi y}{2\delta}\right) = \tau_w \cos\left(\frac{\pi y}{2\delta}\right) = \frac{\tau_w}{2} \text{ when } \frac{\pi y}{2\delta} = \frac{\pi}{3}, \text{ or}: \ y = \frac{2\delta}{3} = \mathbf{0.03\ m}$$

+3