P3.161 Extend Prob. 3.46 to the problem of computing the center of pressure L of the normal face F_n, as in Fig. P3.161. (At the center of pressure, no moments are required to hold the plate at rest.) Neglect friction. Express your result in terms of the sheet thickness h_1 and the angle θ between the plate and the oncoming jet 1.

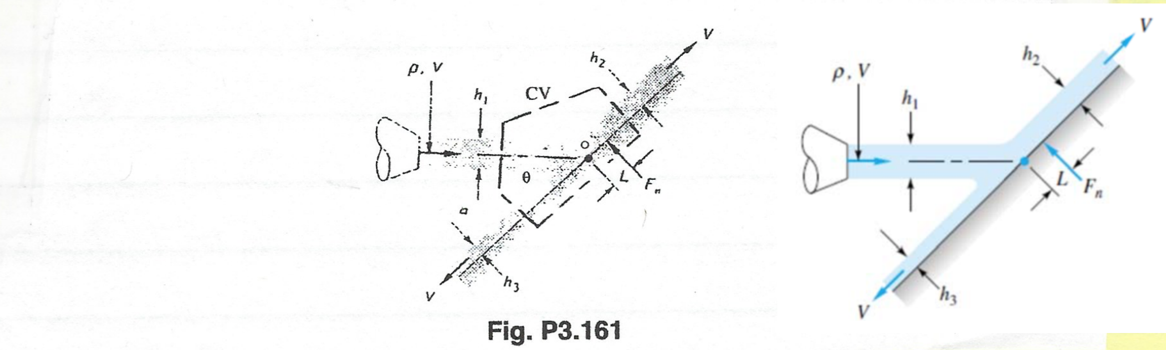

Fig. P3.161

$$\sum M_o = \frac{d}{dt}\int_{CV} (\underline{r}\times\underline{v})\rho\, d\forall + \int_{CS} (\underline{r}\times\underline{v})\rho\, \underline{V}_R\cdot\underline{n}\, dA$$

$$\sum M_o = -F_n L = -\dot m_1 |\underline{r}_1\times\underline{V}|_z + \dot m_2 |\underline{r}_2\times\underline{V}_2|_z + \dot m_3 |\underline{r}_3\times\underline{V}_3|_z$$

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$$-F_n L = \dot m_2 \frac{h_2}{2} V + \dot m_3 \frac{h_3}{2}(-V)$$

$$-\rho V h_1 \cancel{V} \sin\theta\, L = \rho \cancel{V} h_2 \frac{h_2}{2} \cancel{V} + \rho \cancel{V} h_3\left(-\frac{h_3}{2}\cancel{V}\right)$$

$$L = \frac{-(h_2^2 - h_3^2)}{2h_1\sin\theta} = \frac{-h_1^2\cos\theta}{2h_1\sin\theta}$$

$$= -\frac{1}{2} h_1 \cot\theta$$

Continuity: $h_2 = h_1 - h_3$

F_t : $0 = -h_1\cos\theta + h_2 - h_3$

ie $h_3 = h_1/2\,(1-\cos\theta)$

$h_2 = h_1/2\,(1+\cos\theta)$

$\therefore\ h_2^2 - h_3^2 = h_1^2\cos\theta$

$\underline{r} = a\hat{e}_t + s\hat{e}_n$

$\underline{r}\times\underline{V} = (a\hat{e}_t + s\hat{e}_n)\times V\hat{e}_t = sV\hat{e}_s$ s = span wise direction

right hand rule

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