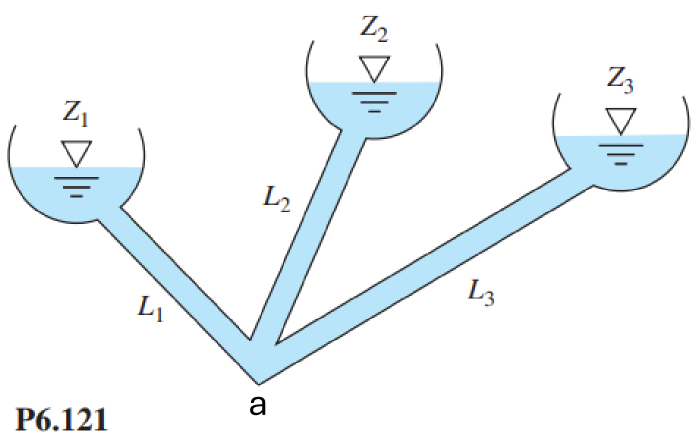
P6.121 Consider the three-reservoir system of Fig. P6.121 with the following data:

$$L_1 = 95\text{m} \quad L_2 = 125\text{ m} \quad L_3 = 160\text{ m}$$

$$z_1 = 25\text{ m} \quad z_2 = 115\text{ m} \quad z_3 = 85\text{ m}$$

All pipes are 28-cm-diameter unfinished concrete ($\varepsilon = 1$ mm). Compute the steady flow rate in all pipes for water at 20°C.



P6.121

$$\rho = 998\, kg/m^3$$

$$\mu = 0.001 \frac{kg}{ms}$$

$$\frac{\varepsilon}{d} = \frac{1}{280} = 0.00357$$

For the three pipes:

$$z_1 - h_a = f_1 \frac{L_1}{d_1} \frac{V_1^2}{2g}$$

$$z_2 - h_a = f_2 \frac{L_2}{d_2} \frac{V_2^2}{2g}$$

$$z_3 - h_a = f_3 \frac{L_3}{d_3} \frac{V_3^2}{2g}$$

Continuity:

$$Q_1 + Q_2 + Q_3 = 0$$

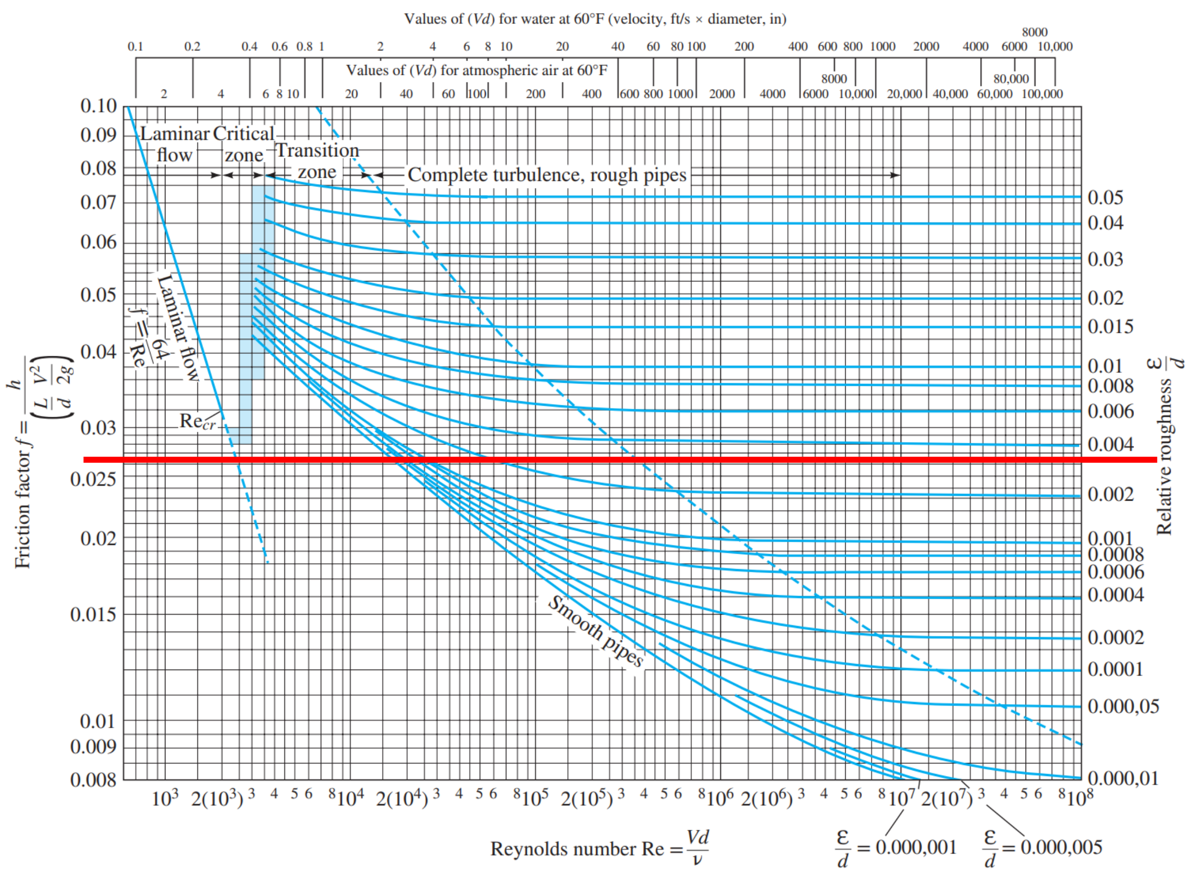
And since diameter is the same:

$$V_1 + V_2 + V_3 = 0$$

Guess h_a, use the average length of the pipes:

$$h_a = \frac{25 + 115 + 85}{3} = 75m$$

Reynolds number is unknown, but $\frac{\varepsilon}{d} = 0.00357 \rightarrow$ enter Moody chart for fully turbulent region.



First guess for friction factor f_i in all pipes: 0.027.

The equations become:

$$25 - 75 = (-)50 = f_1 \frac{L_1}{d_1} \frac{V_1^2}{2g} = 0.027 \frac{95}{0.28} \frac{V_1^2}{2(9.81)} \rightarrow V_1 = -10.35 m/s$$

$$115 - 75 = 40 = f_2 \frac{L_2}{d_2} \frac{V_2^2}{2g} = 0.027 \frac{125}{0.28} \frac{V_2^2}{2(9.81)} \rightarrow V_2 = 8.07 m/s$$

$$85 - 75 = 10 = f_3 \frac{L_3}{d_3} \frac{V_3^2}{2g} = 0.027 \frac{160}{0.28} \frac{V_3^2}{2(9.81)} \rightarrow V_3 = 3.57 m/s$$

Verify continuity:

$$V_1 + V_2 + V_3 = -10.35 + 8.07 + 3.57 = 1.29 \neq 0$$

Repeating for $h_a = 80m$ gives:

$$V_1 = -10.85, V_2 = 7.55, V_3 = 2.52$$

Verify continuity:

$$V_1 + V_2 + V_3 = -0.78$$

Interpolate for $h_a = 78m$:

$$V_1 = -10.65, V_2 = 7.76, V_3 = 2.98$$

Verify continuity:

$$V_1 + V_2 + V_3 = -0.09$$

Tolerance is acceptable, resulting flowrates:

$$Q_1 = -0.66 \frac{m^3}{s}, Q_2 = 0.48 \frac{m^3}{s}, Q_3 = 0.18 \frac{m^3}{s}$$