5.60 A variable mesh screen produces a linear and axisymmetric velocity profile as indicated in Fig. P5.60 in the air flow through a 2-ft-diameter circular cross section duct. The static pressures upstream and downstream of the screen are 0.2 and 0.15 psi and are uniformly distributed over the flow cross section area. Neglecting the force exerted by the duct wall on the flowing air, calculate the screen drag force.

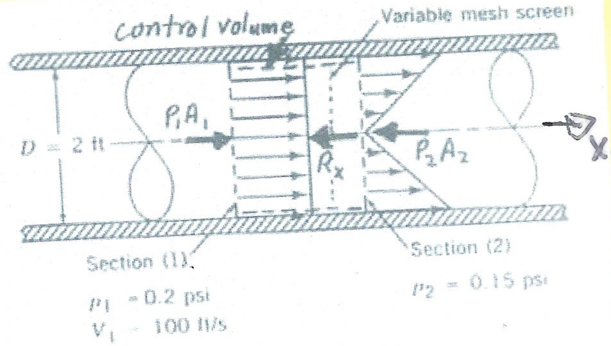

FIGURE P5.60

$$\underline{\Sigma F} = \frac{d}{dt}\int_{cv} \underline{V}\rho\, d\forall + \int_{cs} \underline{V}\rho \underline{V_R}\cdot \underline{n}\, dA$$

x only, steady flow, fixed cv ie $\underline{V_s} = 0$, inlet = 1 outlet = 2

Conservation momentum:

$$p_1A_1 - p_2A_2 - R_x = -V_1\rho V_1 A_1 + \int_0^R u_2\rho u_2 2\pi r\,dr$$

$$R_x = \rho V_1^2 \frac{\pi D_1^2}{4} - 2\pi\rho\int_0^R \left(u_{max}\frac{r}{R}\right)^2 r\,dr + (p_1 - p_2)\frac{\pi D_1^2}{4}$$

Conservation mass:

$$0 = \frac{d}{dt}\int_{cv}\rho\, d\forall + \int_{cs}\rho \underline{V_R}\cdot\underline{n}\, dA$$

$$0 = \int_{cs}\rho\underline{V}\cdot\underline{n}\, dA$$

$$= -\rho V_1 A_1 + \int_0^R \rho u_2 2\pi r\,dr$$

$$\cancel{\rho} V_1 \frac{\cancel{\pi} D_1^2}{4} = \cancel{\rho}\int_0^R \left(u_{max}\frac{r}{R}\right) 2\cancel{\pi} r\,dr$$

$$\frac{V_1 D_1^2}{4} = \frac{2u_{max}}{R}\int_0^R r^2\,dr \qquad \int_0^R r^2\,dr = \left.\frac{r^3}{3}\right|_0^R = \frac{R^3}{3}$$

$$= \frac{2u_{max}}{R}\frac{R^3}{3}$$

$$V_1R^2 = \frac{2u_{max}R^2}{3}$$

$$u_{max} = \frac{3}{2}V_1 = 150 \text{ ft/s}$$

$$\int_0^R \left(u_{max}\frac{r}{R}\right)^2 r\,dr$$

$$= \frac{u_{max}^2}{R^2}\int_0^R r^3\,dr$$

$$= \frac{u_{max}^2}{R^2}\frac{R^4}{4}$$

$$= \frac{u_{max}^2 R^2}{4}$$

$$= \frac{u_{max}^2 D^2}{16}$$

$$R_x = 13.3 \text{ lb}$$

P5.60 A variable mesh screen produces a linear and axisymmetric velocity profile as indicated in Fig P5.60 in the air flow through a 2 *ft* diameter circular cross duct. The static pressure upstream and downstream of the screen are 0.2 and 0.15 psi and are uniformly distributed over the flow cross section area. Neglecting the force exerted by the duct wall on the flowing air, calculate the screen drag force.

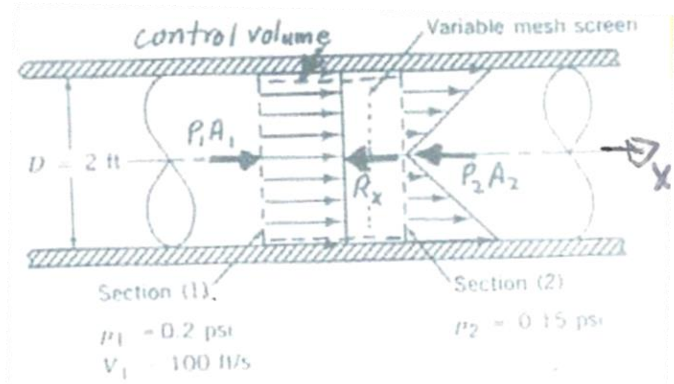

FIGURE P5.60

$$\sum F = \frac{d}{dt}\int_{CV} \underline{V}\rho d\forall + \int_{CS} \underline{V}\rho \underline{V_R} \cdot \underline{n} dA$$

Steady state, fixed CV *ie* $\underline{V_s} = 0$, inlet=1, outlet=2

$$p_1A_1 - p_2A_2 - R_x = -V_1\rho V_1 A_1 + \int_0^R u_2\rho u_2 2\pi r dr$$

$$R_x = \rho V_1^2 \frac{\pi D_1^2}{4} - 2\pi\rho \int_0^R \left(u_{max}\frac{r}{R}\right)^2 r dr + (p_1 - p_2)\frac{\pi D_1^2}{4}$$

$$0 = \frac{d}{dt}\int_{CV} \rho d\forall + \int_{CS} \rho \underline{V_R} \cdot \underline{n} dA$$

$$0 = \int_{CS} \rho \underline{V} \cdot \underline{n} dA$$

$$= -\rho V_1 A_1 + \int_0^R \rho u_2 2\pi r dr$$

$$\cancel{\rho} V_1 \frac{\cancel{\pi} D_1^2}{4} = \cancel{\rho}\int_0^R \left(u_{max}\frac{r}{R}\right) 2\cancel{\pi} r dr$$

$$\frac{V_1 D_1^2}{4} = \frac{2u_{max}}{R}\int_0^R r^2 dr$$

$$= \frac{2u_{max}}{R}\frac{R^3}{3}$$

$$V_1 R^2 = \frac{2u_{max}R^2}{3}$$

$$u_{max} = \frac{3}{2}V_1 = 150\ ft/s$$

$$R_x = 13.3\ lb$$

$$\int_0^R \left(u_{max}\frac{r}{R}\right)^2 r dr$$

$$= \frac{u_{max}^2}{R^2}\int_0^R r^3 dr$$

$$= \frac{u_{max}^2}{R^2}\frac{R^4}{4}$$

$$= \frac{u_{max}^2 R^2}{4}$$

$$= \frac{u_{max}^2 D^2}{16}$$

$$\int_0^R r^2 dr = \frac{r^3}{3}\bigg|_0^R$$

$$= \frac{R^3}{3}$$