3.44 Consider uniform flow past a cylinder with a V-shaped *wake*, as shown. Pressures at (1) and (2) are equal. Let b be the width into the paper. Find a formula for the force F on the cylinder due to the flow. Also compute $C_D = F/(\rho U^2 Lb)$.

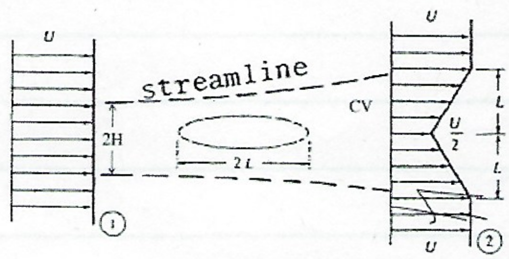

Fig. P3.44

Conservation of mass:

$$0 = \frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho \, \underline{V}_R \cdot \underline{n} \, dA$$

Steady flow, fixed CV

$$0 = \int_{CS} \rho \, \underline{V} \cdot \underline{n} \, dA$$

$$0 = -\int_1 \rho u \, dA + \int_2 \rho u \, dA$$

$$= -\rho U 2H b + 2\rho \int_0^L \frac{U}{2}\left(1 + \frac{y}{L}\right) dy \, b$$

$$= -\rho U 2H b + 2\rho b \underbrace{\left[\frac{U}{2}y + \frac{U}{4L}y^2\right]_0^L}$$

$$2\rho b\left(\frac{UL}{2} + \frac{UL}{4}\right) = \frac{3}{2}\rho b U L$$

$$\rho U 2H b = \frac{3}{2}\rho b U L$$

$$H = \frac{3}{4}L$$

Conservation momentum:

$$\Sigma \underline{F} = \frac{d}{dt}\int_{cv} \rho \underline{v}\, d\forall + \int_{cs} \rho \underline{v}\, \underline{v_R}\cdot\underline{n}\, dA$$

Steady flow, fixed CV, F_{drag} = drag force on cylinder

$p = p_a \Rightarrow \Delta p$ force $= 0$

$= \rho U^2 L b\, C_D$

$C_D = \frac{F_{drag}}{\rho U^2 L b}$

$$\Sigma F_x = -F_{drag} = -\int_1 \rho u^2 dA + \int_2 \rho u^2 dA$$

$$= -\rho U^2 2Hb + 2\rho \underbrace{\int_0^L \left[\frac{U}{2}\left(1+\frac{y}{L}\right)\right]^2 dy\, b}$$

$$2\rho b \int_0^L \frac{U^2}{4}\left(1+\frac{2y}{L}+\frac{y^2}{L^2}\right) dy = 2\rho b\left[\frac{U^2}{4}\left(y+\frac{y^2}{L}+\frac{y^3}{3L^2}\right)\Big|_0^L\right.$$

$$= 2\rho b\left[\frac{U^2}{4}\left(L+L+\frac{L}{3}\right)\right]$$

$$= 2\rho b \frac{U^2}{4}\left(\frac{7L}{3}\right)$$

$$= \frac{7}{6}\rho b U^2 L$$

$$-F_{drag} = -\rho U^2 2Hb + \frac{7}{6}\rho b U^2 L$$

$$= -\rho U^2 2\left(\frac{3L}{4}\right)b + \frac{7}{6}\rho b U^2 L$$

$$= \rho U^2 L b\left[-\frac{3}{2}+\frac{7}{6}\right] = \rho U^2 L b\left(-\frac{1}{3}\right)$$

$$\frac{F_{drag}}{\rho U^2 L b} = C_D = \frac{1}{3}$$