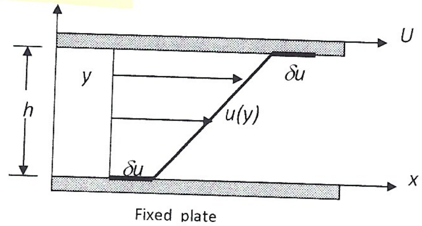
P1.43 For the flow between two parallel plates of Fig. 1.8, reanalyze for the case of *slip flow* at both walls. Use the simple slip condition, $u_{wall} = \ell\,(du/dy)_{wall}$, where ℓ is the mean free path of the fluid. (*a*) Sketch the expected velocity profile and (*b*) find an expression for the shear stress at each wall.

Solution: As in Fig. 1.8, the shear stress remains constant between the two plates. The analysis is correct up to the relation $u = a + by$. There would be equal slip velocities, δu, at both walls, as shown in the following sketch



Ans. (*a*)

$\tau = \mu \frac{du}{dy}$ ie $\frac{du}{dy} = \tau/\mu$ = constant $\Rightarrow u = a + by$

For $\delta u = 0$:

$u(0) = 0 = a$

$u(h) = U = bh$

$b = U/h$

$u = \frac{U}{h} y$

$\tau = \mu \frac{du}{dy} = \mu \frac{U}{h}$

$\delta u_w = 0 \quad K_n = \frac{\lambda}{L} << 1$

$\delta u_w \neq 0 \quad K_n \sim 1$

λ = molecular length scale

L = fluid length scale

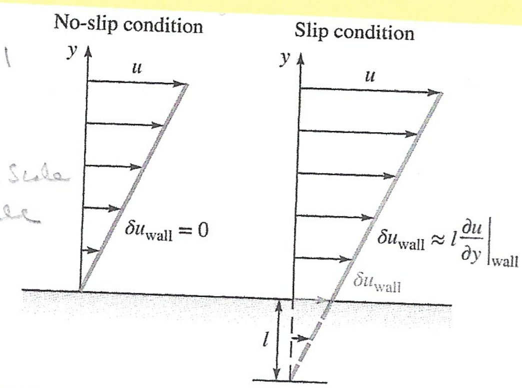


Fig. 1.16 Schematics of velocity profiles over a solid surface with no-slip and slip conditions.

Required, e.g., moving contact line singularity etc.

no slip: $V_f = V_w$ slip: $V_f = l \frac{\partial u}{\partial y}\Big|_w$ l = slip length

$\mu = 0$ can not satisfy $V_f = V_w$ only $V_{nf} = V_{nw}$

Apply boundary conditions:

$u(0) = \delta u = l \frac{\partial u}{\partial y}\Big|_w = a = lb \qquad \frac{\partial u}{\partial y}\Big|_w = b$

$u(h) = U - \delta u = a + bh = U - lb = lb + bh \Rightarrow b = U/(h+2l)$

$u = a + by = lb + by = (l + y)\frac{U}{h+2l} \qquad u(0) = \delta u \quad u(h) = U - \delta u$

$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_w = \frac{\mu U}{h + 2l}$