6.108

6.108 An incompressible Newtonian fluid flows steadily between two infinitely long, concentric cylinders as shown in Fig. P6.108. The outer cylinder is fixed, but the inner cylinder moves with a longitudinal velocity V_0 as shown. The pressure gradient in the axial direction is $-\Delta p/\ell$. For what value of V_0 will the drag on the inner cylinder be zero? Assume that the flow is laminar, axisymmetric, and fully developed.

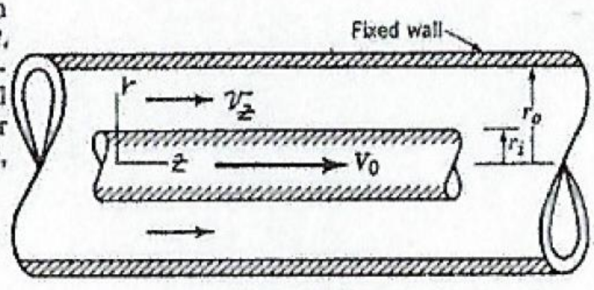

FIGURE P6.108

Equation 6.147, which was developed for flow in circular tubes, applies in the annular region. Thus,

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)r^2 + c_1 \ln r + c_2 \qquad (1)$$

With boundary conditions, $r = r_o$, $v_z = 0$, and $r = r_i$, $v_z = V_0$, it follows that:

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)r_o^2 + c_1 \ln r_o + c_2 \qquad (2)$$

$$V_0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)r_i^2 + c_1 \ln r_i + c_2 \qquad (3)$$

Subtract Eq. (2) from Eq. (3) to obtain

$$V_0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right) + c_1 \ln \frac{r_i}{r_o}$$

so that

$$c_1 = \frac{V_0 - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right)}{\ln \frac{r_i}{r_o}}$$

The drag on the inner cylinder will be zero if

$$\left(\tau_{rz}\right)_{r=r_i} = 0$$

Since, $$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad \text{(Eq. 6.126f)}$$

and with $v_r = 0$, it follows that

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

(con't)

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Differentiate Eq. (1) with respect to r to obtain

$$\frac{\partial v_z}{\partial r} = \frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right)r + \frac{c_1}{r}$$

so that at $r = r_i$

$$\left(\tau_{rz}\right)_{r=r_i} = \mu\left[\frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right)r_i + \frac{V_o - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right)}{r_i \ln\frac{r_i}{r_o}}\right]$$

Thus, in order for the drag to be zero,

$$\frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right)r_i + \frac{V_o - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right)}{r_i \ln\frac{r_i}{r_o}} = 0$$

or

$$\underline{\underline{V_o = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left[2\,r_i^2 \ln\frac{r_i}{r_o} - \left(r_i^2 - r_o^2\right)\right]}}$$