ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW8 Solution

***P5.60** The thrust F of a free propeller, either aircraft or marine, depends upon density ρ, the rotation rate n in r/s, the diameter D, and the forward velocity V. Viscous effects are slight and neglected here. Tests of a 25-cm-diameter model aircraft propeller, in a sea-level wind tunnel, yield the following thrust data at a velocity of 20 m/s:

Rotation rate, r/min	4800	6000	8000
Measured thrust, N	6.1	19	47

(*a*) Use this data to make a crude but effective dimensionless plot. (b) Use the dimensionless data to predict the thrust, in newtons, of a similar 1.6-m-diameter prototype propeller when rotating at 3800 r/min and flying at 225 mi/h at 4000 m standard altitude.

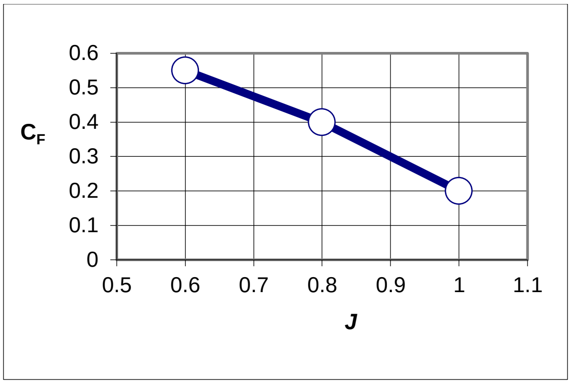
Solution: The given function is $F = \text{fcn}(\rho, n, D, V)$, and we note that $j = 3$. Hence we expect 2 pi groups. The writer chose (ρ, n, D) as repeating variables and found this:

$$C_F = fcn(J), \quad where \quad C_F = \frac{F}{\rho n^2 D^4} \quad and \quad J = \frac{V}{nD}$$

The quantity C_F is called the *thrust coefficient*, while J is called the *advance ratio*. Now use the data (at $\rho = 1.2255$ kg/m^3) to fill out a new table showing the two pi groups:

n, r/s	133.3	100.0	80.0
C_F	0.55	0.40	0.20
J	0.60	0.80	1.00

A crude but effective plot of this data is as follows. *Ans.*(*a*)



(*b*) At 4000 m altitude, from Table A.6, $\rho = 0.8191\ kg/m^3$. Convert 225 mi/h = 101.6 m/s. Convert 3800 r/min = 63.3 r/s. Then find the prototype advance ratio:

$$J = (101.6\ m/s)/[(63.3\ r/s)(1.6\ m)] = 1.00$$

Well, lucky us, that's our third data point! Therefore $C_{F,prototype} \approx 0.20$. And the thrust is

$$F_{prototype} = C_F \rho n^2 D^4 = (0.20)(0.8191\frac{kg}{m^3})(63.3\frac{r}{s})^2(1.6m)^4 \approx \mathbf{4300\ N} \quad Ans.(b)$$

***P5.76** A 2-ft-long model of a ship is tested in a freshwater tow tank. The measured drag may be split into "friction" drag (Reynolds scaling) and "wave" drag (Froude scaling). The model data are as follows:

Tow speed, ft/s:	0.8	1.6	2.4	3.2	4.0	4.8
Friction drag, lbf:	0.016	0.057	0.122	0.208	0.315	0.441
Wave drag, lbf:	0.002	0.021	0.083	0.253	0.509	0.697

The prototype ship is 150 ft long. Estimate its total drag when cruising at 15 kn in seawater at 20°C.

Solution: For fresh water at 20°C, take $\rho = 1.94$ slug/ft^3, $\mu = 2.09E{-}5$ slug/ft·s. Then evaluate the Reynolds numbers and the Froude numbers and respective force coefficients:

Vm, ft/s:	0.8	1.6	2.4	3.2	4.0	4.8
Rem = VmLm/ν:	143000	297000	446000	594000	743000	892000
$C_{F,friction}$:	0.00322	0.00287	0.00273	0.00261	0.00254	0.0024 7

For seawater, take $\rho = 1.99$ slug/ft^3, $\mu = 2.23E{-}5$ slug/ft·s. With Lp = 150 ft and Vp = 15 knots = 25.3 ft/s, evaluate

$$Re_{proto} = \frac{\rho_p V_p L_p}{\mu_p} = \frac{1.99(25.3)(150)}{2.23E{-}5} \approx 3.39E8; \quad Fr_p = \frac{25.3}{[32.2(150)]^{1/2}} \approx 0.364$$

For $\quad Fr \approx 0.364, \quad$ interpolate to $C_{F,wave} \approx 0.0027$

Thus we can immediately estimate Fwave $\approx 0.0027(1.99)(25.3)^2(150)^2 \approx$ 77000 lbf. However, as mentioned in Fig. 5.8 of the text, **Rep is far outside the range of the friction force data**, therefore we must *extrapolate* as best we can. A power-law curve-fit is

$$C_{F,friction} \approx \frac{0.0178}{Re^{0.144}}, \quad \text{hence } C_{F,proto} \approx \frac{0.0178}{(3.39E8)^{0.144}} \approx 0.00105$$

$$\frac{Q_m}{Q_p} = \frac{V_m}{V_p}\frac{A_m}{A_p} = \sqrt{\frac{L_m}{L_p}}\left(\frac{L_m}{L_p}\right)^2 = \left(\frac{L_m}{L_p}\right)^{5/2} \text{ or } \alpha^{5/2}$$

$$Fig.5/9: \alpha = 1:65; \quad \therefore\ Q_{model} = (10100\frac{ft^3}{s})(\frac{1}{65})^{5/2} = 0.30\frac{ft^3}{s} \quad Ans.$$

Thus Ffriction $\approx 0.00105(1.99)(25.3)^2(150)^2 \approx$ 30000 lbf. **Ftotal ≈ 107000 lbf**. *Ans.*

P6.3 The Keystone Pipeline in the chapter opener photo has a maximum proposed flow rate of 1.3 million barrels of crude oil per day. Estimate the Reynolds number and whether the flow is laminar. Assume that Keystone crude oil fits Fig. A.1 of the Appendix at 40ºC.

Solution: From Fig. A.1 of the Appendix, for crude oil at 40ºC, ρ = (SG)ρ_{water} = 0.86(1000) = 860 kg/m^3, and $\mu \approx$ 0.0054 kg/m·s. (a) Convert 1,300,000 barrels per day to 2.39 m^3/s (Appendix C) and a diameter of 36 in equals 0.914 m. Then the Reynolds number is

$$\text{Re}_d = \frac{\rho V d}{\mu} = \frac{4\rho Q}{\pi \mu d} = \frac{4(860 kg/m^3)(2.39 m^3/s)}{\pi(0.0054 kg/m \cdot s)(0.914 m)} = 530{,}000$$

The flow is definitely *turbulent*. *Ans*.

P6.24 Two tanks of water at 20°C are connected by a capillary tube 4 mm in diameter and 3.5 m long. The surface of tank 1 is 30 cm higher than the surface of tank 2.
(a) Estimate the flow rate in m^3/h. Is the flow laminar? (b) For what tube diameter will Re_d be 500?

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. (a) Both tank surfaces are at atmospheric pressure and have negligible velocity. The energy equation, when neglecting minor losses, reduces to:

$$\Delta z = 0.3\ m = h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.001\ kg/m{\cdot}s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)(0.004\ m)^4}$$

$$\text{Solve for}\quad Q = 5.3E\text{–}6\ \frac{m^3}{s} = \mathbf{0.019}\ \frac{\mathbf{m^3}}{\mathbf{h}}\quad \textit{Ans.}\ \text{(a)}$$

Check $\quad Re_d = 4\rho Q/(\pi\mu d) = 4(998)(5.3E\text{–}6)/[\pi(0.001)(0.004)]$

$\mathbf{Re_d = 1675}$ ***laminar.*** *Ans.* (a)

(b) If $Re_d = 500 = 4\rho Q/(\pi\mu d)$ and $\Delta z = h_f$, we can solve for both Q and d:

$$Re_d = 500 = \frac{4(998\ kg/m^3)Q}{\pi(0.001\ kg/m{\cdot}s)d},\quad \textbf{\textit{or}}\quad Q = 0.000394d$$

$$h_f = 0.3\ m = \frac{128(0.001\ kg/m{\cdot}s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)d^4},\quad or\quad Q = 20600d^4$$

Combine these two to solve for $Q = 1.05E\text{–}6\ m^3/s$ *and* **d = 2.67 mm** *Ans.* (b)

P6.25 For the configuration shown in Fig. P6.25, the fluid is ethyl alcohol at 20°C, and the tanks are very wide. Find the flow rate that occurs, in m^3/h. Is the flow laminar?

Solution: For ethanol, take $\rho = 789$ kg/m^3 and $\mu = 0.0012$ kg/m·s. Write the energy equation from upper free surface (1) to lower free surface (2):

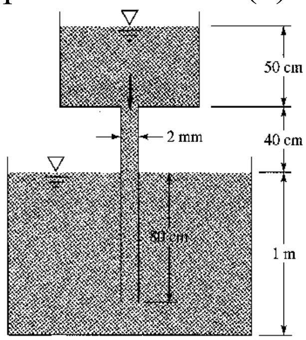


Fig. P6.25

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f, \quad \text{with } p_1 = p_2 \text{ and } V_1 \approx V_2 \approx 0$$

$$\text{Then} \quad h_f = z_1 - z_2 = 0.9 \text{ m} = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.0012)(1.2 \text{ m})Q}{\pi(789)(9.81)(0.002)^4}$$

Solve for $Q \approx 1.90\text{E–}6 \text{ m}^3/\text{s} = \mathbf{0.00684\ m^3/h.}$ *Ans.*

Check the Reynolds number $Re = 4\rho Q/(\pi\mu d) \approx 795$ − **OK, laminar flow.**

C5.5 Does an automobile radio antenna vibrate in resonance due to vortex shedding? Consider an antenna of length *L* and diameter *D*. According to beam-vibration theory [e.g. Kelly [31], p. 401], the first mode natural frequency of a solid circular cantilever beam is $\omega_n = 3.516[EI/(\rho AL^4)]^{1/2}$, where *E* is the modulus of elasticity, *I* is the area moment of inertia, ρ is the beam material density, and *A* is the beam cross-section area. (a) Show that ω_n is proportional to the antenna radius *R*. (b) If the antenna is steel, with $L = 60$ cm and $D = 4$ mm, estimate the natural vibration frequency, in Hz. (c) Compare with the shedding frequency if the car moves at 65 mi/h.

Solution: **(a) From Fig. 2.13 for a circular cross-section, $A = \pi R^2$ and $I = \pi R^4/4$. Then the natural frequency is predicted to be:**

$$\omega_n = 3.516\sqrt{\frac{E\pi R^4/4}{\rho\pi R^2 L^4}} = \mathbf{1.758}\sqrt{\frac{\mathbf{E}}{\boldsymbol{\rho}}\frac{\mathbf{R}}{\mathbf{L^2}}} = Const \times RP \quad Ans.\text{ (a)}$$

(b) For steel, $E = 2.1E11$ Pa and $\rho = 7840$ kg/m^3. If L = 60 cm and D = 4 mm, then

$$\omega_n = 1.758\sqrt{\frac{2.1E11}{7840}\frac{0.002}{0.6^2}} \approx 51\ \frac{rad}{s} \approx \mathbf{8\ Hz} \quad Ans.\text{ (b)}$$

(c) For U = 65 mi/h = 29.1 m/s and sea-level air, check $Re_D = \rho UD/\mu = 1.2(29.1)(0.004)/(0.000018) \approx 7800$. From Fig. 5.2*b*, read Strouhal number St ≈ 0.21. Then,

$$\frac{\omega_{shed}D}{2\pi U} = \frac{\omega_{shed}(0.004)}{2\pi(29.1)} \approx 0.21, \quad or: \quad \omega_{shed} \approx 9600\ \frac{rad}{s} \approx \mathbf{1500\ Hz} \quad Ans.\text{ (c)}$$

Thus, for a typical antenna, the shedding frequency is far higher than the natural vibration frequency.