ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW6 Solution

P4.2 Flow through the converging nozzle in Fig. P4.2 can be approximated by the one-dimensional velocity distribution

$$u \approx V_o\left(1+\frac{2x}{L}\right) \quad \upsilon \approx 0 \quad w \approx 0$$

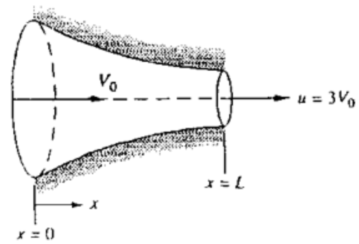


Fig. P4.2

(a) Find a general expression for the fluid acceleration in the nozzle. (b) For the specific case V_o = 10 ft/s and L = 6 in, compute the acceleration, in g's, at the entrance and at the exit.

Solution: Here we have only the single 'one-dimensional' convective acceleration:

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} = \left[V_o\left(1+\frac{2x}{L}\right)\right]\frac{2V_o}{L} = \frac{\mathbf{2V_o^2}}{\mathbf{L}}\left(\mathbf{1+\frac{2x}{L}}\right) \quad \textit{Ans.}\text{ (a)}$$

$$\textit{For } L = 6'' \textit{ and } V_o = 10\ \frac{ft}{s},\quad \frac{du}{dt} = \frac{2(10)^2}{6/12}\left(1+\frac{2x}{6/12}\right) = 400(1+4x),\ \textit{with x in feet}$$

At x = 0, du/dt = **400** ft/s^2 (12 g's); at x = L = 0.5 ft, du/dt = **1200** ft/s^2 (37 g's). *Ans.* (b)

P4.27 A frictionless, incompressible steady-flow field is given by

$$\mathbf{V} = 2xy\mathbf{i} - y^2\mathbf{j}$$

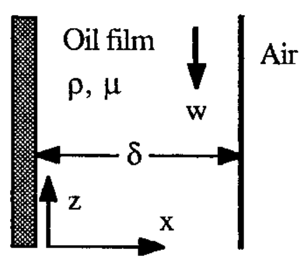
in arbitrary units. Let the density be ρ_o = constant and neglect gravity. Find an expression for the pressure gradient in the x direction.

Solution: For this (gravity-free) velocity, the momentum equation is

$$\rho\left(u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}\right) = -\nabla p, \quad \text{or:} \quad \rho_o[(2xy)(2y\mathbf{i}) + (-y^2)(2x\mathbf{i} - 2y\mathbf{j})] = -\nabla p$$

$$\text{Solve for} \quad \nabla p = -\rho_o(2xy^2\mathbf{i} + 2y^3\mathbf{j}), \quad \text{or:} \quad \boldsymbol{\frac{\partial p}{\partial x} = -\rho_o 2xy^2} \quad \textit{Ans.}$$

P4.80 An oil film drains steadily down the side of a vertical wall, as shown. After an initial development at the top of the wall, the film becomes independent of z and of constant thickness. Assume that w = w(x) only that the atmosphere offers no shear resistance to the film. (a) Solve Navier-Stokes for w(x). (b) Suppose that film thickness and $[\partial w/\partial x]$ at the wall are measured. Find an expression which relates μ to this slope $[\partial w/\partial x]$.



Solution: First, there is no pressure gradient $\partial p/\partial z$ because of the constant-pressure atmosphere. The Navier-Stokes z-component is $\mu(d^2w/dx^2) = \rho g$, and the solution requires w = 0 at x = 0 and (dw/dx) = 0 (no shear at the film edge) at x = δ. The solution is:

$$\boldsymbol{w = \frac{\rho g x}{2\mu}(x - 2\delta)} \quad \textit{Ans.} \text{ (a)} \quad \text{NOTE: } w \text{ is negative (down)}$$

The wall slope is $dw/dx|_{wall} = -\rho g\delta/\mu$, rearrange: $\mu = -\rho g\delta/[dw/dx|_{wall}]$ *Ans.* (b)

P4.36 A constant-thickness film of viscous liquid flows in laminar motion down a plate inclined at angle θ, as in Fig. P4.36. The velocity profile is

$$u = Cy(2h - y) \quad v = w = 0$$

(a) Find the constant *C* in terms of the specific weight and viscosity and the angle θ. (b) Find the volume flow rate *Q* per unit width in terms of these parameters.

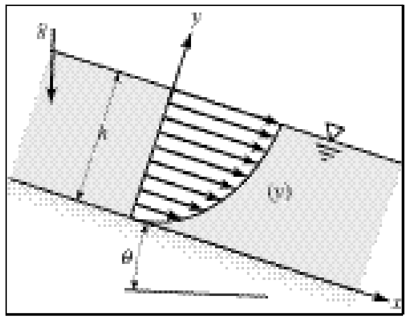


Fig. P4.36

Solution: There is atmospheric pressure all along the surface at y = h, hence $\partial p/\partial x = 0$. The x-momentum equation can easily be evaluated from the known velocity profile:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\nabla^2 u, \quad \text{or:} \quad 0 = 0 + \rho g \sin\theta + \mu(-2C)$$

$$\text{Solve for} \quad \mathbf{C = \frac{\rho g \sin\theta}{2\mu}} \quad \textit{Ans.} \text{ (a)}$$

The flow rate per unit width is found by integrating the velocity profile and using C:

$$Q = \int_0^h u\,dy = \int_0^h Cy(2h - y)\,dy = \frac{2}{3}Ch^3 = \mathbf{\frac{\rho g h^3 \sin\theta}{3\mu}} \text{ per unit width} \quad \textit{Ans.} \text{ (b)}$$

P4.88 The viscous oil in Fig. P4.88 is set into steady motion by a concentric inner cylinder moving axially at velocity *U* inside a fixed outer cylinder. Assuming constant pressure and density and a purely axial fluid motion, solve Eqs. (4.38) for the fluid velocity distribution $v_z(r)$. What are the proper boundary conditions?

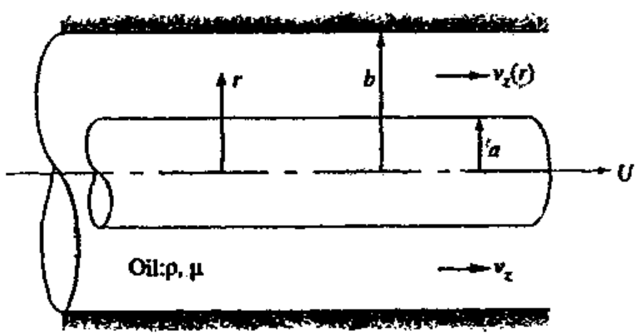


Fig. P4.88

Solution: If vz = fcn(r) only, the *z*-momentum equation (Appendix E) reduces to:

$$\rho\frac{dv_z}{dt} = -\frac{\partial p}{\partial z} + \rho g_z + \mu\nabla^2 v_z, \quad \text{or:} \quad 0 = 0 + 0 + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

The solution is $v_z = C_1 \ln(r) + C_2$, subject to $v_z(a) = U$ and $v_z(b) = 0$

Solve for $C_1 = U/\ln(a/b)$ and $C_2 = -C_1 \ln(b)$

The final solution is: $$\mathbf{v_z = U\frac{\ln(r/b)}{\ln(a/b)}}$$ *Ans.*

C4.2 A belt moves upward at velocity V, dragging a film of viscous liquid of thickness h, as in Fig. C4.2. Near the belt, the film moves upward due to no-slip. At its outer edge, the film moves downward due to gravity. Assuming that the only non-zero velocity is $v(x)$, with zero shear stress at the outer film edge, derive a formula for (a) $v(x)$; (b) the average velocity V_{avg} in the film; and (c) the wall velocity V_C for which there is no net flow either up or down.
(d) Sketch $v(x)$ for case (c).

Solution: (a) The assumption of parallel flow, $u = w = 0$ and $v = v(x)$, satisfies continuity and makes the x- and z-momentum equations irrelevant. We are left with the y-momentum equation:

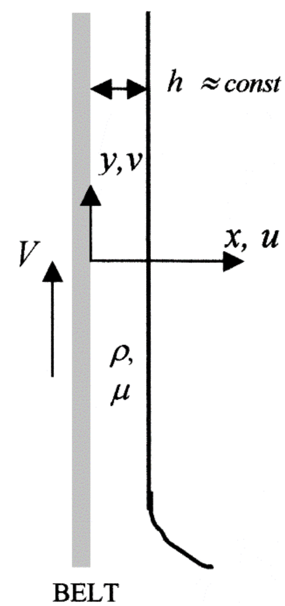


Fig. C4.2

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}-\rho g+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

There is no convective acceleration, and the pressure gradient is negligible due to the free surface. We are left with a second-order linear differential equation for $v(x)$:

$$\frac{d^2 v}{dx^2}=\frac{\rho g}{\mu} \quad \text{Integrate: } \frac{dv}{dx}=\frac{\rho g}{\mu}x+C_1 \quad \text{Integrate again: } v=\frac{\rho g}{\mu}\frac{x^2}{2}+C_1 x+C_2$$

At the free surface, $x = h$, $\tau = \mu(dv/dx) = 0$, hence $C_1 = -\rho gh/\mu$. At the wall, $v = V = C_2$. The solution is

$$\boldsymbol{v = V - \frac{\rho g h}{\mu}x + \frac{\rho g}{2\mu}x^2} \quad \textit{Ans.} \text{ (a)}$$

(b) The average velocity is found by integrating the distribution $v(x)$ across the film:

$$v_{avg} = \frac{1}{h}\int_0^h v(x)\,dx = \frac{1}{h}\left[Vx - \frac{\rho g h x^2}{2\mu} + \frac{\rho g x^3}{6\mu}\right]_0^h = \boldsymbol{V - \frac{\rho g h^2}{3\mu}} \quad \textit{Ans.}\ \text{(b)}$$

(c) Since $hv_{avg} \equiv Q$ per unit depth into the paper, there is no net up-or-down flow when $\boldsymbol{V = \rho g h^2/(3\mu)}$ *Ans.* (c)

(d) A graph of case (c) is shown below. *Ans.* (d)