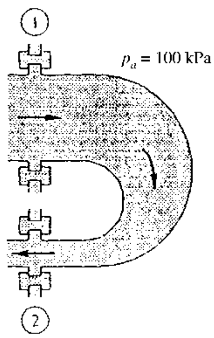
ME:5160 (58:160) Intermediate Mechanics of Fluids
Fall 2024 – HW5 Solution

P3.77 Water at 20°C flows steadily through a reducing pipe bend, as in Fig. P3.77. Known conditions are $p1 = 350$ kPa, $D1 = 25$ cm, $V1 = 2.2$ m/s, $p2 = 120$ kPa, and $D2 = 8$ cm. Neglecting bend and water weight, estimate the total force which must be resisted by the flange bolts.



Solution: First establish the mass flow and exit velocity:

$$\dot{m} = \rho_1 A_1 V_1 = 998\left(\frac{\pi}{4}\right)(0.25)^2(2.2) = 108\ \frac{\text{kg}}{\text{s}} = 998\left(\frac{\pi}{4}\right)(0.08)^2 V_2, \quad \text{or} \quad V_2 = 21.5\ \frac{\text{m}}{\text{s}}$$

The CV surrounds the bend and cuts through the flanges. The force balance is

$$\sum F_x = -F_{\text{bolts}} + p_{1,\text{gage}}A_1 + p_{2,\text{gage}}A_2 = \dot{m}_2 u_2 - \dot{m}_1 u_1, \quad \text{where } u_2 = -V_2 \quad \text{and} \quad u_1 = V_1$$

$$\text{or} \quad F_{\text{bolts}} = (350000 - 100000)\frac{\pi}{4}(0.25)^2 + (120000 - 100000)\frac{\pi}{4}(0.08)^2 + 108(21.5 + 2.2)$$

$$= 12271 + 101 + 2553 \approx \mathbf{14900\ N} \quad \textit{Ans.}$$

P3.94 A water jet 3 inches in diameter strikes a concrete (SG = 2.3) slab which rests freely on a level floor. If the slab is 1 ft wide into the paper, calculate the jet velocity which will just begin to tip the slab over.

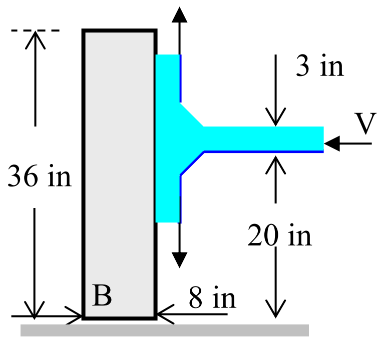


Fig. P3.94

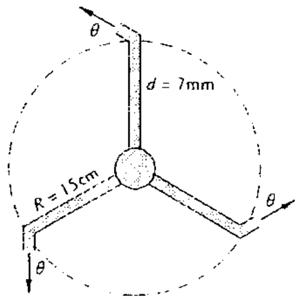
Solution: For water let ρ = 1.94 slug/ft^3. Find the water force and then take moments about the lower left corner of the slab, point B. A control volume around the water flow yields

$$\sum F_x = F_{on\ jet} = \Sigma\, m_{out} u_{out} - \Sigma\, m_{in} u_{in} = m_{out}(0) - \rho A V(-V),\ \ F = \rho A V^2$$

$$\sum M_B = (\rho A V^2)\left(\frac{21.5}{12} ft\right) - W_{slab}\left(\frac{4}{12} ft\right),\ W_{slab} = (2.3 \times 62.4)\left(\frac{8}{12} ft\right)(3\,ft)(1\,ft) = 287\ lbf$$

Thus $(1.94)\frac{\pi}{4}\left(\frac{3}{12} ft\right)^2 V^2\left(\frac{21.5}{12} ft\right) = (287\,lbf)\left(\frac{4}{12} ft\right)$, solve for $V_{jet} = \mathbf{23.7\frac{ft}{s}}$ *Ans.*

P3.153 The 3-arm lawn sprinkler of Fig. P3.153 receives 20°C water through the center at 2.7 m^3/hr. If collar friction is neglected, what is the steady rotation rate in rev/min for (a) $\theta = 0°$; (b) $\theta = 40°$?



Solution: The velocity exiting each arm is

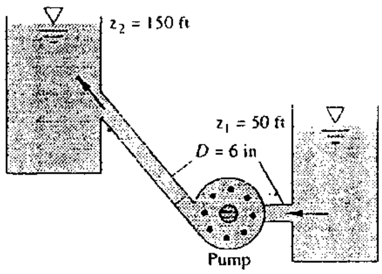
$$V_o = \frac{Q/3}{(\pi/4)d^2} = \frac{2.7/[(3600)(3)]}{(\pi/4)(0.007)^2} = 6.50\ \frac{m}{s}$$

With negligible air drag and bearing friction, the steady rotation rate (Example 3.15) is

$$\omega_{final} = \frac{V_o \cos\theta}{R} \quad \text{(a) } \theta = 0°: \quad \omega = \frac{(6.50)\cos 0°}{0.15\ m} = 43.3\ \frac{rad}{s} = \mathbf{414}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans. (a)}$$

$$\text{(b) } \theta = 40°: \quad \omega = \omega_o \cos\theta = (414)\cos 40° = \mathbf{317}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans. (b)}$$

P3.180 Water at 20°C is pumped at 1500 gal/ min from the lower to the upper reservoir, as in Fig. P3.180. Pipe friction losses are approximated by $hf \approx 27V^2/(2g)$, where V is the average velocity in the pipe. If the pump is 75 percent efficient, what horse-power is needed to drive it?



Solution: First evaluate the average velocity in the pipe and the friction head loss:

$$Q = \frac{1500}{448.8} = 3.34\ \frac{ft^3}{s}, \quad \text{so } V = \frac{Q}{A} = \frac{3.34}{\pi(3/12)^2} = 17.0\ \frac{ft}{s} \quad \text{and} \quad h_f = 27\frac{(17.0)^2}{2(32.2)} \approx \mathbf{121\ ft}$$

Then apply the steady flow energy equation:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:} \quad 0 + 0 + 50 = 0 + 0 + 150 + 121 - h_p$$

$$\text{Thus } h_p = 221\ \text{ft}, \quad \text{so } P_{pump} = \frac{\gamma Q h_p}{\eta} = \frac{(62.4)(3.34)(221)}{0.75}$$

$$= 61600\ \frac{ft \cdot lbf}{s} \approx \mathbf{112\ hp} \quad \textit{Ans.}$$

P3.183 The pump in Fig. P3.183 creates a 20°C water jet oriented to travel a maximum horizontal distance. System friction head losses are 6.5 m. The jet may be approximated by the trajectory of friction-less particles. What power must be delivered by the pump?

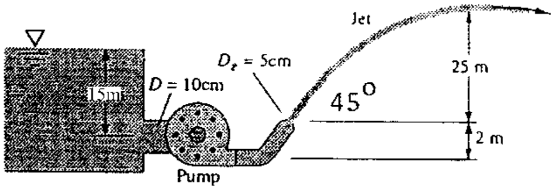


Fig. P3.183

Solution: For maximum travel, the jet must exit at $\theta = 45°$, and the exit velocity must be

$$V_2 \sin\theta = \sqrt{2g\Delta z_{max}} \quad \text{or:} \quad V_2 = \frac{[2(9.81)(25)]^{1/2}}{\sin 45°} \approx 31.32 \ \frac{m}{s}$$

The steady flow energy equation for the piping system may then be evaluated:

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_f - h_p,$$

$$\text{or:} \quad 0 + 0 + 15 = 0 + (31.32)^2/[2(9.81)] + 2 + 6.5 - h_p, \quad \text{solve for } h_p \approx 43.5 \text{ m}$$

$$\text{Then} \quad P_{pump} = \gamma Q h_p = (9790)\left[\frac{\pi}{4}(0.05)^2(31.32)\right](43.5) \approx \mathbf{26200\ W} \quad \textit{Ans.}$$