ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW2 Solution

P1.73 A small submersible moves at velocity V in 20° C water at 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is Ca ≈ 0.25. At what velocity will cavitation bubbles form? Will the body cavitate if V = 30 m/s and the water is cold (5° C)?

Solution: From Table A-5 at 20° C read pv = 2.337 kPa. By definition,

$$Ca_{crit} = 0.25 = \frac{2(p_a - p_v)}{\rho V^2} = \frac{2(131000 - 2337)}{(998\ \text{kg/m}^3)V^2}, \quad \text{solve } V_{crit} \approx \mathbf{32.1\ m/s} \quad \textit{Ans.}\ \text{(a)}$$

If we decrease water temperature to 5° C, the vapor pressure reduces to 863 Pa, and the density changes slightly, to 1000 kg/m^3. For this condition, if V = 30 m/s, we compute:

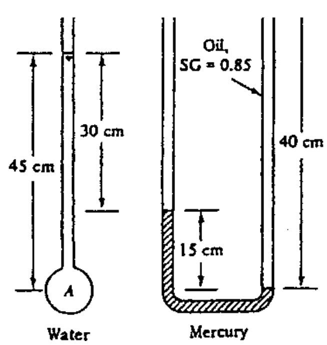
$$Ca = \frac{2(131000 - 863)}{(1000)(30)^2} \approx 0.289$$

This is *greater* than 0.25, therefore the body **will not cavitate for these conditions**. *Ans.* (b)

P2.45 Determine the gage pressure at point A in Fig. P2.45, in pascals. Is it higher or lower than Patmosphere?

Solution: Take γ = 9790 N/m^3 for water and 133100 N/m^3 for mercury. Write the hydrostatic formula between the atmosphere and point A:

$$\begin{aligned} p_{atm} &+ (0.85)(9790)(0.4\ \text{m}) \\ &- (133100)(0.15\ \text{m}) - (12)(0.30\ \text{m}) \\ &+ (9790)(0.45\ \text{m}) = p_A, \end{aligned}$$



or: $p_A = p_{atm} - 12200\ \text{Pa} = \mathbf{12200\ Pa\ (vacuum)}$ *Ans.*

P2.77 Circular gate ABC is hinged at B. Compute the force just sufficient to keep the gate from opening when h = 8 m. Neglect atmospheric pressure.

Solution: The hydrostatic force on the gate is

$$F = \gamma h_{CG} A = (9790)(8\text{ m})(\pi\text{ m}^2)$$
$$= 246050\text{ N}$$

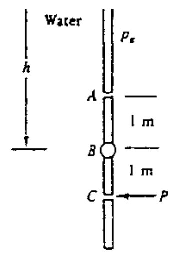


Fig. P2.77

This force acts below point B by the distance

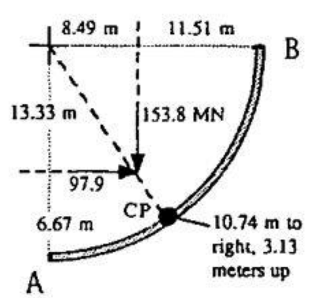
$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(\pi/4)(1)^4\sin 90^\circ}{(8)(\pi)} = -0.03125\text{ m}$$

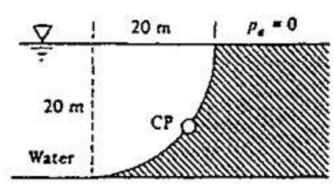
Summing moments about B gives P(1 m) = (246050)(0.03125 m), or $P \approx$ **7690 N** *Ans.*

P2.82 The dam in Fig. P2.82 is a quarter-circle 50 m wide into the paper. Determine the horizontal and vertical components of hydrostatic force against the dam and the point CP where the resultant strikes the dam.

Solution: The horizontal force acts as if the dam were vertical and 20 m high:

$$F_H = \gamma h_{CG} A_{vert} = \left(9790\frac{N}{m^3}\right)(10\,m)(20 \times 50\,m^2) = \mathbf{97.9\,MN}\ Ans.$$





C1.11 Mott [Ref. 49, p. 38] discusses a simple falling-ball viscometer, which we can analyze later in Chapter 7. A small ball of diameter *D* and density ρ_b falls through a tube of test liquid. The fall velocity *V* is calculated by the time to fall a measured distance. The formula for calculating the viscosity of the fluid is

$$\mu = \frac{(\rho_b - \rho)\, g\, D^2}{18\, V}$$

This result is limited by the requirement that the Reynolds number ($\rho VD/\mu$) be less than 1.0. Suppose a steel ball (SG = 7.87) of diameter 2.2 mm falls in SAE 25W oil (SG = 0.88) at 20°C. The measured fall velocity is 8.4 cm/s. (*a*) What is the viscosity of the oil, in kg/m-s?
(*b*) Is the Reynolds number small enough for a valid estimate?

Solution: Relating SG to water, Eq. (1.7), the steel density is 7.87(1000) = 7870 kg/m^3 and the oil density is 0.88(1000) = 880 kg/m^3. Using SI units, the formula predicts

$$\mu_{oil} = \frac{(7870 - 880\, kg/m^3)(9.81\, m/s^2)(0.0022\, m)^2}{18(0.084\, m/s)} \approx \mathbf{0.22\ \frac{kg}{m-s}} \quad \textit{Ans.}$$

$$\text{Check} \quad \text{Re} = \frac{\rho VD}{\mu} = \frac{(880)(0.084)(0.0022)}{0.22} = 0.74 < 1.0 \quad \textit{OK}$$

As mentioned, we shall analyze this falling sphere problem in Chapter 7.