ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW10 Solution

P6.73 For 20ºC water flow in a smooth, horizontal 10-cm pipe, with $\Delta p/L$ = 1000 Pa/m, the writer computed a flow rate of 0.030 m^3/s. (a) Verify, or disprove, the writer's answer. (b) If verified, use the power-law friction factor relation, Eq. 6.41, to estimate the pipe diameter that will triple this flow rate. (c) For extra credit, use the more exact friction factor relation, Eq. (6.38), to solve part (b).

Solution: (a) For water at 20ºC, ρ = 998 kg/m^3, and μ = 0.0010 kg/m·s. The pressure-drop relation is

$$\Delta p / L = 1000 = \frac{f}{d}\left(\frac{\rho V^2}{2}\right) = \frac{f}{0.1}\left(\frac{998 V^2}{2}\right), \; or \;\; V^2 \approx \frac{0.2004}{f} \quad (\text{SI units})$$

$$\frac{1}{\sqrt{f}} = 2.0\log_{10}(\mathrm{Re}_d\sqrt{f}) - 0.8 \;\; ; \;\; \mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{(998)(0.1)V}{0.0010}$$

This is ideal for Excel iteration: Guess $f \approx 0.020$, get $V \approx 3.17$ m/s, $\mathrm{Re}_d \approx 316{,}000$. Repeat: $f \approx 0.0138$, get $V \approx 3.81$ m/s, $\mathrm{Re}_d \approx 380{,}000$. Once more: $f \approx 0.01384$, get $V \approx$ 3.805 m/s,

$\mathrm{Re}_d \approx 379{,}700$. CONVERGED: $V \approx 3.805$ m/s, $Q = (\pi/4)d^2V$ = **0.030** m^3/s. Writer verified!

(b) Eq. (6.41) predicts that

$$Q^{1.75} \propto d^{4.75}, \text{ or } \; d = const\, Q^{0.368}$$

$$\text{If } \; Q_2 = 3Q_1 \;, \text{ then } d_2 = d_1 (3)^{0.368} = 1.50\, d_1 = 1.50(0.1) \approx \; \mathbf{0.15\,m} \qquad Ans(b)$$

(c) Raise Q to 3(0.030) = 0.090 m^3/s, and use EES to find the new diameter for the same $\Delta p/L$. The more exact answer is $\boldsymbol{d_2}$ **= 0.1514 m**, corresponding to $\mathrm{Re}_2 = 753{,}000$. The power-law result (b) is quite accurate, considering that Eq. (6.41) is recommended only for $\mathrm{Re}_d \le 100{,}000$.

P6.80 The head-versus-flowrate characteristics of a centrifugal pump are shown in Fig. P6.80. If this pump drives water at 20°C through 120 m of 30-cm-diameter cast-iron pipe, what will be the resulting flow rate, in m^3/s?

Solution: For water, take $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m·s. For cast iron, take $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.26/300 \approx 0.000867$. The head loss must match the pump head:

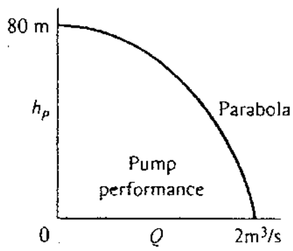

Fig. P6.80

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = \frac{8fLQ^2}{\pi^2 g d^5} = h_{pump} \approx 80 - 20Q^2, \quad \text{with Q in } m^3/s$$

$$\text{Evaluate} \quad h_f = \frac{8f(120)Q^2}{\pi^2(9.81)(0.3)^5} = 80 - 20Q^2, \quad \text{or:} \quad Q^2 \approx \frac{80}{20 + 4080f}$$

$$\text{Guess} \quad f \approx 0.02, \quad Q = \left[\frac{80}{20 + 4080(0.02)}\right]^{1/2} \approx 0.887\ \frac{m^3}{s}, \quad Re = \frac{4\rho Q}{\pi\mu d} \approx 3.76E6$$

$$\frac{\varepsilon}{d} = 0.000867, \quad f_{better} \approx 0.0191, \quad Re_{better} \approx 3.83E6, \quad \text{converges to } \mathbf{Q \approx 0.905\ \frac{m^3}{s}} \quad \textit{Ans.}$$

P6.98 A rectangular heat exchanger is to be divided into smaller sections using sheets of commercial steel 0.4 mm thick, as sketched in Fig. P6.98. The flow rate is 20 kg/s of water at 20°C. Basic dimensions are L = 1 m, W = 20 cm, and H = 10 cm. What is the proper number of *square* sections if the overall pressure drop is to be no more than 1600 Pa?

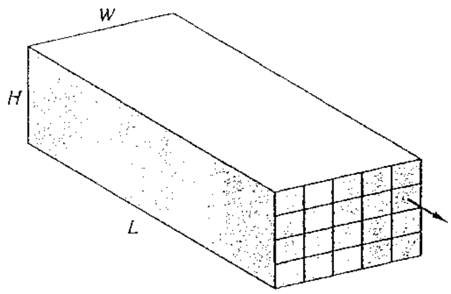

Fig. P6.98

Solution: For water at 20°C, take *r* = 998 kg/m^3 and *m* = 0.001 kg/m×s. For commercial steel, *e* » 0.046 mm. Let the short side (10 cm) be divided into "J" squares. Then the long (20 cm) side divides into "2J" squares and altogether there are N = 2J^2 squares. Denote the side length of the square as "a," which equals (10 cm)/J minus the wall thickness. The hydraulic diameter of a square exactly equals its side length, Dh = *a*. The total cross-section area is A = N a^2. Then the pressure drop relation becomes

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 = f\frac{1.0}{a}\left(\frac{998}{2}\right)\left(\frac{Q}{Na^2}\right)^2 \le 1600 \text{ Pa}, \quad \text{where} \quad N = 2J^2 \text{ and } a = \frac{0.1}{J} - 0.0004$$

As a first estimate, neglect the 0.4-mm wall thickness, so *a* » 0.1/J. Then the relation for Dp above reduces to fJ » 0.32. Since f » 0.036 for this turbulent Reynolds number (Re » 1E4) we estimate that J » 9 and in fact this is not bad even including wall thickness:

$$J = 9, \quad N = 2(9)^2 = 162, \quad a = \frac{0.1}{9} - 0.0004 = 0.0107 \text{ m}, \quad V = \frac{20/998}{162(0.0107)^2} \approx 1.078 \ \frac{m}{s}$$

$$Re = \frac{\rho V a}{\mu} = \frac{998(1.078)(0.0107)}{0.001} \approx 11526, \quad \frac{\varepsilon}{a} = \frac{0.046}{10.7} \approx 0.00429, \quad f_{Moody} \approx 0.0360$$

$$\text{Then} \quad \Delta p = (0.036)\left(\frac{1.0}{0.0107}\right)\left(\frac{998}{2}\right)(1.078)^2 \approx 1950 \text{ Pa}$$

So the wall thickness increases V and decreases a so Dp is too large. Try J = 8:

$$J = 8, \quad N = 128, \quad a = 0.0121 \text{ m}, \quad V = 1.069 \ \frac{m}{s},$$

$$Re = 12913, \quad \frac{\varepsilon}{a} = 0.0038, \quad f \approx 0.0347$$

$$\text{Then} \quad \Delta p = f(L/a)(\rho/2)V^2 \approx \mathbf{1636\ Pa}. \quad \text{Close enough,} \ J = 8, \ \mathbf{N = 128} \quad \textit{Ans.}$$

[I suppose a practical person would specify J = 7, N = 98, to keep Dp < 1600 Pa.]

***P6.102** A 70 percent efficient pump delivers water at 20°C from one reservoir to another 20 ft higher, as in Fig. P6.102. The piping system consists of 60 ft of galvanized-iron 2-in pipe, a reentrant entrance, two screwed 90° long-radius elbows, a screwed-open gate valve, and a sharp exit. What is the input power required in horsepower with and without a 6° well-designed conical expansion added to the exit? The flow rate is 0.4 ft^3/s.

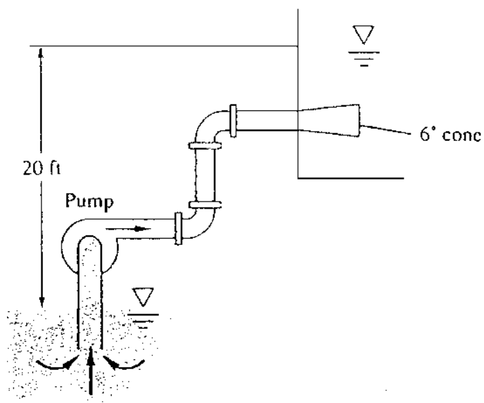

Fig. P6.102

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For galvanized iron, $\varepsilon \approx$ 0.0005 ft, whence ε/d = 0.0005/(2/12 ft) ≈ 0.003. Without the 6° cone, the minor losses are:

$$K_{\text{reentrant}} \approx 1.0; \quad K_{\text{elbows}} \approx 2(0.41); \quad K_{\text{gate valve}} \approx 0.16; \quad K_{\text{sharp exit}} \approx 1.0$$

$$\text{Evaluate} \quad V = \frac{Q}{A} = \frac{0.4}{\pi(2/12)^2/4} = 18.3 \ \frac{ft}{s}; \quad Re = \frac{\rho V d}{\mu} = \frac{1.94(18.3)(2/12)}{2.09E{-}5} \approx 284000$$

At this Re and roughness ratio, we find from the Moody chart that f ~ 0.0266. Then

$$\text{(a)}\quad h_{pump} = \Delta z + \frac{V^2}{2g}\left(f\frac{L}{d} + \sum K\right) = 20 + \frac{(18.3)^2}{2(32.2)}\left[0.0266\left(\frac{60}{2/12}\right) + 1.0 + 0.82 + 0.16 + 1.0\right]$$

$$\text{or}\quad h_{pump} \approx 85.6\text{ ft},\quad \text{Power} = \frac{\rho g Q h_p}{\eta} = \frac{(62.4)(0.4)(85.6)}{0.70}$$

$$= 3052 \div 550 \approx \mathbf{5.55\ hp}\quad \textit{Ans.}\text{ (a)}$$

(b) If we replace the sharp exit by a 6° conical diffuser, from Fig. 6.23, K_{exit} ~ 0.3. Then

$$h_p = 20 + \frac{(18.3)^2}{2(32.2)}\left[0.0266\left(\frac{60}{2/12}\right) + 1.0 + .82 + .16 + 0.3\right] = 81.95\text{ ft}$$

$$\text{then}\quad \text{Power} = (62.4)(0.4)(81.95)/0.7 \div 550 \approx \mathbf{5.31\ hp}\ \text{(4\% less)}\quad \textit{Ans.}\text{ (b)}$$

P6.115 In Fig. P6.115 all pipes are 8-cm-diameter cast iron. Determine the flow rate from reservoir (1) if valve C is (a) closed; and (b) open, with $K_{valve} = 0.5$.

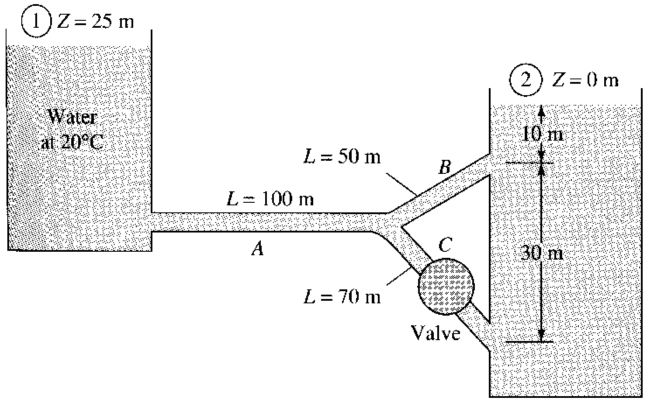

Fig. P6.115

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon \approx$ 0.26 mm, hence $\varepsilon/d = 0.26/80 \approx 0.00325$ for all three pipes. Note $p_1 = p_2$, $V_1 = V_2 \approx 0$. These are long pipes, but we might wish to account for minor losses anyway:

sharp entrance at A: $K_1 \approx 0.5$; line junction from A to B: $K_2 \approx 0.9$ (Table 6.5)

branch junction from A to C: $K_3 \approx 1.3$; two submerged exits: $K_B = K_C \approx 1.0$

If valve C is closed, we have a straight *series* path through A and B, with the same flow rate Q, velocity V, and friction factor *f* in each. The energy equation yields

$$z_1 - z_2 = h_{fA} + \sum h_{mA} + h_{fB} + \sum h_{mB},$$

$$\text{or:}\quad 25\text{ m} = \frac{V^2}{2(9.81)}\left[f\,\frac{100}{0.08} + 0.5 + 0.9 + f\,\frac{50}{0.08} + 1.0\right], \quad \text{where } f = \text{fcn}\left(\text{Re}, \frac{\varepsilon}{d}\right)$$

Guess f ≈ f $_{\text{fully rough}}$ ≈ 0.027, then V ≈ 3.04 m/s, Re ≈ 998(3.04)(0.08)/(0.001) ≈ 243000, ε/d = 0.00325, then f ≈ 0.0273 (converged). Then the velocity through A and B is V = 3.03 m/s, and Q = (π/4)(0.08)^2(3.03) ≈ **0.0152 m^3/s**. *Ans.* (a).

If valve C is open, we have parallel flow through B and C, with QA = QB + QC and, with *d* constant, VA = VB + VC. The total head loss is the same for paths A-B and A-C:

$$z_1 - z_2 = h_{fA} + \sum h_{mA\text{-}B} + h_{fB} + \sum h_{mB} = h_{fA} + \sum h_{mA\text{-}C} + h_{fC} + \sum h_{mC},$$

$$\text{or:}\quad 25 = \frac{V_A^2}{2(9.81)}\left[f_A\frac{100}{0.08} + 0.5 + 0.9\right] + \frac{V_B^2}{2(9.81)}\left[f_B\frac{50}{0.08} + 1.0\right]$$

$$= \frac{V_A^2}{2(9.81)}\left[f_A\frac{100}{0.08} + 0.5 + 1.3\right] + \frac{V_C^2}{2(9.81)}\left[f_C\frac{70}{0.08} + 1.0\right]$$

plus the additional relation VA = VB + VC. Guess f ≈ ffully rough ≈ 0.027 for all three pipes and begin. The initial numbers work out to

$$2g(25) = 490.5 = V_A^2(1250f_A + 1.4) + V_B^2(625f_B + 1) = V_A^2(1250f_A + 1.8) + V_C^2(875f_C + 1)$$

$$\text{If } f \approx 0.027,\ \text{solve (laboriously) } V_A \approx 3.48\text{ m/s},\ V_B \approx 1.91\text{ m/s},\ V_C \approx 1.57\text{ m/s}.$$

$$\text{Compute } Re_A = 278000,\quad f_A \approx 0.0272,\quad Re_B = 153000,\quad f_B = 0.0276,$$
$$Re_C = 125000,\quad f_C = 0.0278$$

Repeat once for convergence: VA ≈ 3.46 m/s, VB ≈ 1.90 m/s, VC ≈ 1.56 m/s. The flow rate from reservoir (1) is **QA** = (π/4)(0.08)^2(3.46) ≈ **0.0174 m^3/s**. (14% more) *Ans.* (b)

C6.4 Suppose you build a house out in the 'boonies,' where you need to run a pipe to the nearest water supply, which fortunately is about 1 km above the elevation of your house. The gage pressure at the water supply is 1 MPa. You require a minimum of 3 gal/min when your end of the pipe is open to the atmosphere. To minimize cost, you want to buy the smallest possible diameter pipe with an extremely smooth surface.
(a) Find the total head loss from pipe inlet to exit, neglecting minor losses.
(b) Which is more important to this problem, the head loss due to elevation difference, or the head loss due to pressure drop in the pipe?
(c) Find the minimum required pipe diameter.

Solution: Convert 3.0 gal/min to 1.89E–4 m^3/s. Let 1 be the inlet and 2 be the outlet and write the steady-flow energy equation:

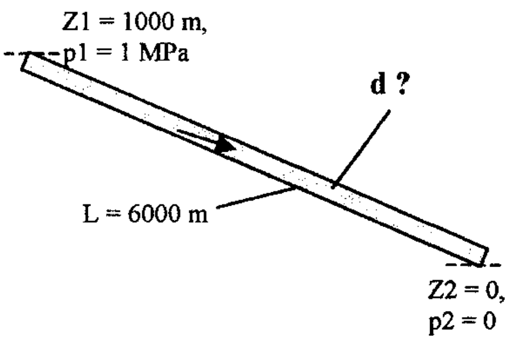

Fig. C6.4

$$\frac{p_{1gage}}{\rho g} + \cancel{\frac{\alpha_1 V_1^2}{2g}} + z_1 = \cancel{\frac{p_{2gage}}{\rho g}} + \cancel{\frac{\alpha_2 V_2^2}{2g}} + z_2 + h_f$$

or: $$h_f = z_1 - z_2 + \frac{p_{1gage}}{\rho g} = 1000\text{ m} + \frac{1E6\text{ kPa}}{998(9.81)} = 1000 + 102 = 1102\text{ m} \quad \textit{Ans.}\text{ (a)}$$

(b) Thus, *elevation drop* of 1000 m is more important to head loss than $\Delta p/\rho g = 102$ m.

(c) To find the minimum diameter, iterate between flow rate and the Moody chart:

$$h_f = f\frac{L}{d}\frac{V^2}{2g}, \quad L = 6000 \text{ m}, \quad \frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right), \quad V = \frac{Q}{\pi d^2/4},$$

$$Q = 1.89\text{E–}4 \ \frac{\text{m}^3}{\text{s}}, \quad \text{Re} = \frac{Vd}{\nu}$$

We are given hf = 1102 m and ν_{water} = 1.005E−6 m^2/s. We can iterate, if necessary with Excel, which can swiftly arrive at the final result:

$$\text{f}_{\text{smooth}} = 0.0266; \quad \text{Re} = 17924; \quad \text{V} = 1.346 \text{ m/s}; \quad \mathbf{d_{min} = 0.0134\ m} \quad \textit{Ans.}\ \text{(c)}$$