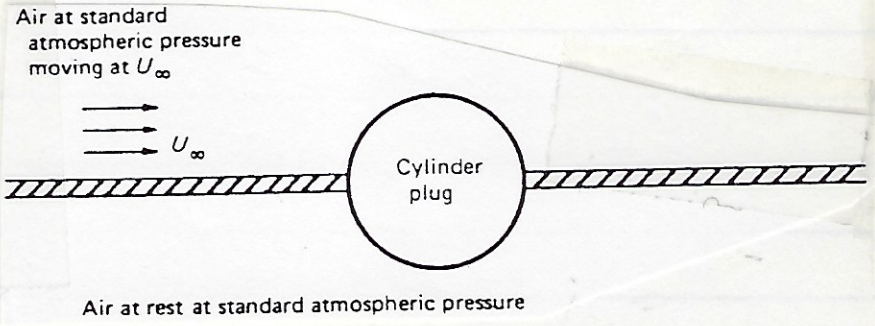
An infinitely long cylinder is used to "plug" a slot in a flat plate, as shown below. Consider the flow of the air to be steady, incompressible, and inviscid. The free-stream atmospheric conditions are

$$P_\infty = 1.01325 \times 10^5 \text{ N/m}^2$$

$$\rho_\infty = 1.2250 \text{ kg/m}^3$$

The cylinder is 7.5 cm in diameter and is made of a material whose density (ρ_c) is 98.00 kg/m^3. What is the minimum velocity of the undisturbed free stream that will lift the cylinder off the plate (neglecting any effects of friction between the plug and the plate)?



$$\int \sin\theta d\theta = -\cos\theta$$

$$\int \sin^3\theta d\theta = -\frac{1}{3}\cos\theta(\sin^2\theta + 2)$$

$$= -\cos\theta + \frac{1}{3}\cos^3\theta$$

$L > W$ is when cylinder will lift off the plate

permit spam $L = -\int_0^\pi (p - p_\infty) r \sin\theta \, d\theta$ $\quad C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2}$

$= 1 - 4\sin^2\theta$

$$= -\tfrac{1}{2}\rho U_\infty^2 R \int_0^\pi C_p \sin\theta \, d\theta$$

$$= -\tfrac{1}{2}\rho U_\infty^2 R \left[-\cos\theta \Big|_0^\pi + 4\left(-\cos\theta \Big|_0^\pi + \tfrac{1}{3}\cos^3\theta \Big|_0^\pi \right) \right]$$

$2 \qquad 2 \qquad -2/3$

$$= \tfrac{1}{2}\rho U_\infty^2 R \{ -2 + 8 - 8/3 \} \qquad \tfrac{18}{3} - \tfrac{8}{3} = \tfrac{10}{3}$$

$$= \tfrac{1}{2}\rho U_\infty^2 R \left(\tfrac{10}{3}\right) = .0766\, U_\infty^2$$

$R = .0375$

$\frac{10}{3} \times \frac{1}{2} \times 1.225 \times U_\infty^2 \times .0375 = W = \rho g A = 9.81 \times 98 \times \frac{\pi}{4}(.075)^2$

permit spam $= 4.25$

$U_\infty^2 = 4.25/.0766$

$U_\infty = 7.447$ m/s