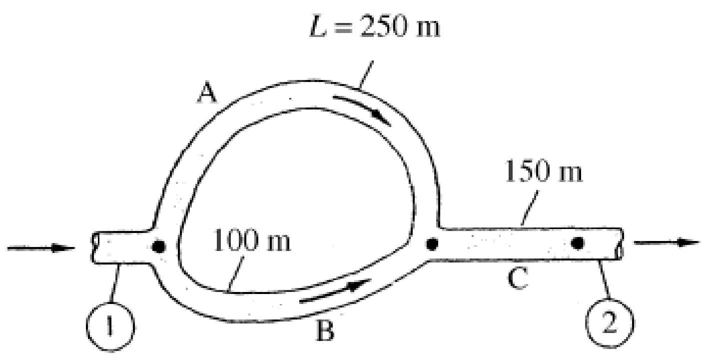
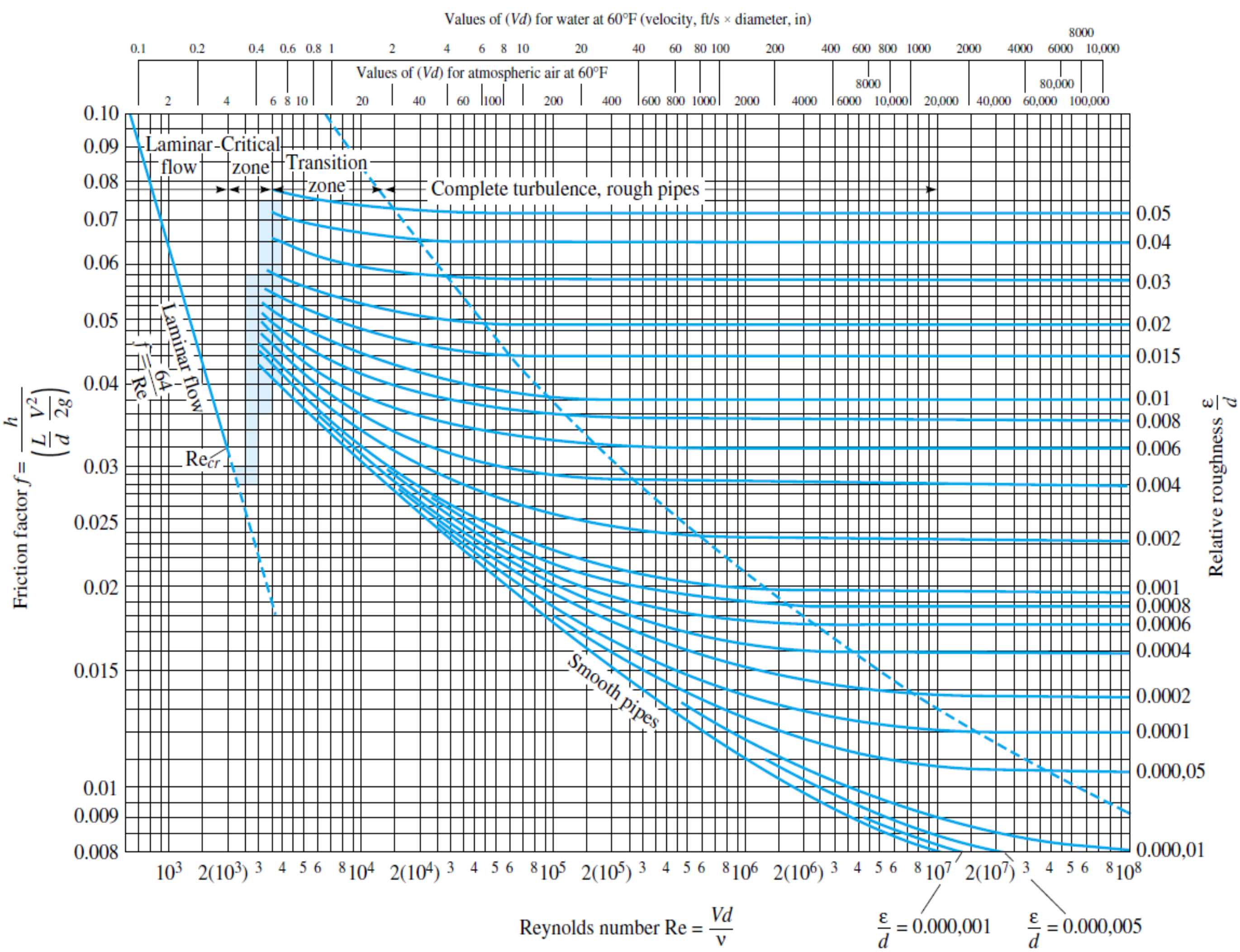
For the horizontal series-parallel system of the Figure below, all pipes are 8-cm-diameter asphalted cast iron (ε = 0.12 mm) filled with water at 20°C (ρ = 998 kg/m^3, μ = 0.001 kg/ms). The total flow rate is Q = 0.0269 m^3/s, the lengths of segments A and C are L_A = 250 m and L_C = 150 m, and velocity in segment A is V_A = 2.06 m/s. Neglect minor losses and find (a) the length of segment B (L_B), and (b) the total pressure drop ($p_1 - p_2$).





1. Solution

(a) Continuity:

$$Q = Q_A + Q_B = V_A\left(\frac{\pi}{4}d^2\right) + V_B\left(\frac{\pi}{4}d^2\right)$$

$$V_B = \frac{Q}{\frac{\pi}{4}d^2} - V_A = \frac{(0.0269)}{\frac{\pi}{4}(0.08)^2} - (2.06) = 3.29\ m/s$$

$$V_C = \frac{Q}{\frac{\pi}{4}d^2} = \frac{(0.0269)}{\frac{\pi}{4}(0.08)^2} = 5.35\ m/s$$

For segment A, B and C:

$$\frac{\varepsilon}{d} = \frac{\left(\frac{0.12}{1000}\right)}{(0.08)} = 0.0015$$

$$(Re_d)_A = \frac{\rho V_A d}{\mu} = \frac{(998)(2.06)(0.08)}{(0.001)} = 164{,}470 \rightarrow\ f_A = 0.0229$$

$$(Re_d)_B = \frac{\rho V_B d}{\mu} = \frac{(998)(3.29)(0.08)}{(0.001)} = 262{,}674 \rightarrow f_B = 0.0225$$

$$(Re_d)_C = \frac{\rho V_C d}{\mu} = \frac{(998)(5.35)(0.08)}{(0.001)} = 427{,}144 \rightarrow f_C = 0.0222$$

For parallel segments A and B the head loss is the same:

$$(h_f)_A = (h_f)_B \rightarrow \left(f\frac{L}{d}\frac{V^2}{2g}\right)_A = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_B$$

$$L_B = L_A\frac{f_A}{f_B}\left(\frac{V_A}{V_B}\right)^2 = (250)\frac{(0.0229)}{(0.0225)}\left(\frac{(2.06)}{(3.29)}\right)^2 = 99.76\ m$$

(b) The energy equation between points (1) and (2) through segment A yields:

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2+\left(h_f\right)_A+\left(h_f\right)_C$$

Since $V_1=V_2$; $z_1=z_2$

$$(p_1-p_2)=\rho g\left[\left(h_f\right)_A+\left(h_f\right)_C\right]=\rho g\left[\left(f\frac{L}{d}\frac{V^2}{2g}\right)_A+\left(f\frac{L}{d}\frac{V^2}{2g}\right)_C\right]$$

$$(p_1-p_2)=(998)(9.81)\left[(0.0229)\frac{(250)}{(0.08)}\frac{(2.06)^2}{2(9.81)}+(0.0222)\frac{(150)}{(0.08)}\frac{(5.35)^2}{2(9.81)}\right]=746{,}052\ Pa$$

Alternatively, through segment B:

$$(p_1-p_2)=\rho g\left[\left(h_f\right)_B+\left(h_f\right)_C\right]=(998)(9.81)\left[(0.0225)\frac{(99.76)}{(0.08)}\frac{(3.29)^2}{2(9.81)}+(0.0222)\frac{(150)}{(0.08)}\frac{(5.35)^2}{2(9.81)}\right]=746{,}059\ Pa$$