8.74 A positive line vortex K is trapped in a corner, as in Fig. P8.74. Compute the total induced velocity at point B, (x, y) = (2a, a), and compare with the induced velocity when no walls are present.

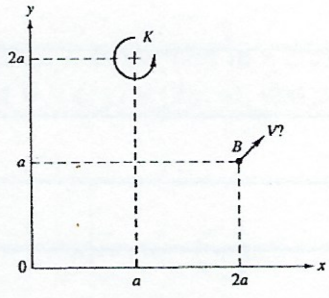
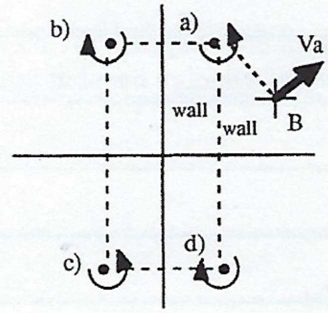


Fig. P8.74



Two walls can be represented by images of the original vortex named as *a* respect to x and y which introduce three more vortices named as b,c,d. The velocity at point B is superposition of the velocity of all four vortices.

For each vortex:

$$\psi = -K \ln r = -K \ln\left(x^2 + y^2\right)^{0.5} ; \ \phi = K\theta = K \tan^{-1}(y/x)$$

$$v_x = \frac{\partial \psi}{\partial y} = \frac{-Ky}{(x^2 + y^2)}$$

$$v_y = -\frac{\partial \psi}{\partial x} = \frac{Kx}{(x^2 + y^2)}$$

For vortex a: Coordinates of point B respect to vortex a is x=a and y=-a and K>0

$$v_x = -\frac{K(-a)}{2a^2} = \frac{K}{2a}; v_y = \frac{Ka}{2a^2} = \frac{K}{2a}$$

For vortex b: Coordinates of point B respect to vortex b is x=3a and y=-a and K<0

$$v_x = -\frac{-K(-a)}{10a^2} = -\frac{K}{10a}; v_y = \frac{-K3a}{10a^2} = -\frac{3K}{10a}$$

For vortex c: Coordinates of point B respect to vortex c is x=3a and y=3a and K>0

$$v_x = -\frac{K(3a)}{18a^2} = -\frac{K}{6a}; v_y = \frac{K3a}{18a^2} = \frac{K}{6a}$$

For vortex d: Coordinates of point B respect to vortex d is x=a and y=3a and K<0

$$v_x = -\frac{-K(3a)}{10a^2} = \frac{3K}{10a}; v_y = \frac{-Ka}{10a^2} = -\frac{K}{10a}$$

$$v_{xB} = \frac{K}{2a} - \frac{K}{10a} - \frac{K}{6a} + \frac{3K}{10a} = \frac{8K}{15a}$$

$$v_{yB} = \frac{K}{2a} - \frac{3K}{10a} + \frac{K}{6a} - \frac{K}{10a} = \frac{4K}{15a}$$

$$V_B = \sqrt{v_{xB}^2 + v_{yB}^2} = 0.59\frac{K}{a}$$

when walls are present

When no walls are present:

$$V_B = V_a = \sqrt{v_{xa}^2 + v_{ya}^2} = \sqrt{\left(\frac{K}{2a}\right)^2 + \left(\frac{K}{2a}\right)^2} = \frac{K}{\sqrt{2}a} = 0.71\frac{K}{a}$$