**8.31** A Rankine half-body is formed as shown in Fig. P8.31. For the conditions shown, compute (a) the source strength $m$ in $m^2/s$; (b) the distance $a$; (c) the distance $h$; and (d) the total velocity at point A.

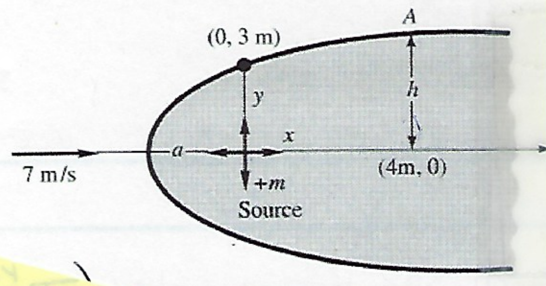


Fig. P8.31

$\psi$ = uniform stream + source

$$\psi(r,\theta) = \underbrace{U r\sin\theta}_{y} + m\theta$$

$x = r\cos\theta$

$y = r\sin\theta$

$\theta = \tan^{-1} y/x$

$\psi(r_s, \theta) = \text{constant}$

$\psi(r_s, \pi) = \pm m\pi$    + upper, − lower

(a) $r_s = m(\pi-\theta)/U\sin\theta$

$$y\Big|_{\substack{x=0\\ \theta=\pi/2}} = 3 = \pi m/2U \Rightarrow m = 13.4 \ m^2/s \qquad U = 7 m/s$$

(b) $u = \dfrac{\partial\psi}{\partial y} = U + \dfrac{m}{r}\cos\theta$     $\dfrac{\partial}{\partial y}(\tan^{-1} y/x) = \dfrac{1}{1+(y/x)^2}\cdot\dfrac{1}{x}$

$u(\pi) = 0 = U - \dfrac{m}{a} \Rightarrow m = Ua$     $= x/(x^2+y^2)$

$a = m/U = 1.91 \ m$     $= \dfrac{r\cos\theta}{r^2} = \dfrac{\cos\theta}{r}$

(c) $r_s = m(\pi-\theta)/U\sin\theta = 4/\cos\theta$ @ $x = 4$

$$\frac{13.4\,(\pi-\theta)}{7\sin\theta} = \frac{4}{\cos\theta}$$

$$13.4/28 = \tan\theta/(\pi-\theta)$$

$\tan x = \dfrac{13.4}{28}(\pi - x)$

using series expansion $= x + \dfrac{x^3}{3} + \cdots$

$x^3 + x(4.436) - 4.51 = 0$

$x = .8688$ rad $= 49.8°$     $r = 4/\cos\theta = 5.95$ m

Solution manual 47.8°     $h = r\sin\theta = 4.41$ m

(d) $v = -\psi_x = \dfrac{m}{r}\sin\theta = -m\dfrac{\partial}{\partial x}(\tan^{-1} y/x)$

$$V = [u^2+v^2]^{1/2} = \underbrace{\left[\left(U + \frac{m}{r}\cos\theta\right)^2 + \left(\frac{m}{r}\sin\theta\right)^2\right]^{1/2}}_{U^2(1 + a^2/r^2 + \frac{2a}{r}\cos\theta)} = 8.7 \ m/s$$

$U^2 + \dfrac{m^2}{r^2}\cos^2\theta + \dfrac{2Um}{r}\cos\theta + \dfrac{m^2}{r^2}\sin^2\theta$