**8.14** A tornado may be modeled as the circulating flow shown in Fig. P8.14, with $\upsilon_r = \upsilon_z = 0$ and $\upsilon_\theta(r)$ such that

$$\upsilon_\theta = \begin{cases} \omega r & r \le R \\ \dfrac{\omega R^2}{r} & r > R \end{cases}$$

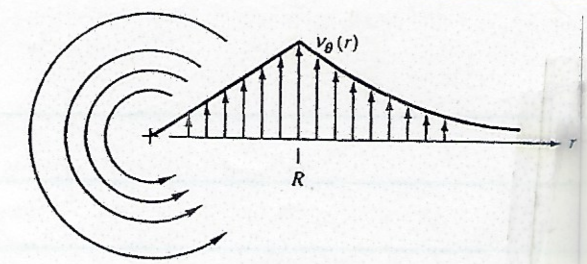


**Fig. P8.14**

Determine whether this flow pattern is irrotational in either the inner or outer region. Using the $r$-momentum equation (D.5) of App. D, determine the pressure distribution $p(r)$ in the tornado, assuming $p = p_\infty$ as $r \to \infty$. Find the location and magnitude of the lowest pressure.

$$\omega_z = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)$$

inner $v_\theta = \omega r$ $\quad \omega_z = 2\omega$

outer $v_\theta = \dfrac{\omega R^2}{r}$ $\quad \omega_z = 0$

$r$ momentum: $$-\frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \Rightarrow \frac{dp}{dr} = \frac{\rho v_\theta^2}{r}$$

$$p_i = \int \frac{\rho}{r}(\omega r)^2 dr = \frac{\rho \omega^2 r^2}{2} + c_i$$

$\int x^n dx = x^{n+1}/n+1$

$$p_o = \int \frac{\rho}{r}\left(\frac{\omega R^2}{r}\right)^2 dr = -\frac{\rho \omega^2 R^4}{2r^2} + c_o$$

$$p_o(r \to \infty) = p_\infty \Rightarrow c_o = p_\infty$$

$$p_i(R) = \frac{\rho \omega^2 R^2}{2} + c_i = p_o(R) = \frac{-\rho \omega^2 R^2}{2} + p_\infty$$

$$c_i = p_\infty - \rho \omega^2 R^2$$

$$p_i(r) = \frac{\rho \omega^2 r^2}{2} + p_\infty - \rho \omega^2 R^2$$

$$p_i(0) = p_\infty - \rho \omega^2 R^2$$