6.90 A 90-ft-long sheet-steel duct carries air at approximately 20°C and 1 atm. The duct cross section is an equilateral triangle whose side measures 9 in. If a blower can supply 1 hp to the flow, what flow rate, in ft^3/s, will result?

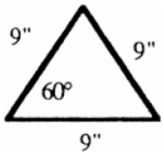

Fig. P6.90

Solution: For air at 20°C and 1 atm, take $\rho \approx 0.00234$ slug/ft^3 and $\mu = 3.76E{-}7$ slug/ft·s. Compute the hydraulic diameter, and express the head loss in terms of Q:

$$D_h = \frac{4A}{P} = \frac{4(1/2)(9)(9\sin 60°)}{3(9)} = 5.2'' = 0.433 \text{ ft}$$

$$h_f = f\frac{L}{D_h}\frac{(Q/A)^2}{2g} = f\left(\frac{90}{0.433}\right)\frac{\{Q/[0.5(9/12)^2 \sin 60°]\}^2}{2(32.2)} \approx 54.4fQ^2$$

Energy Equation

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} + h_p = \cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_z} + h_L$$

$$h_p = h_L = f\frac{L}{D}\frac{V^2}{2g}$$

$$\frac{\dot{w}_p}{\gamma Q} = f\frac{L}{D}\frac{V^2}{2g}$$

$$\dot{w}_p = \gamma Q\, f\frac{L}{D}\frac{V^2}{2g}$$

For sheet steel, take $\varepsilon \approx 0.00015$ ft, hence $\varepsilon/Dh \approx 0.000346$. Now relate everything to the input power:

$$\text{Power} = 1 \text{ hp} = 550\ \frac{\text{ft·lbf}}{\text{s}} = \rho g Q h_f = (0.00234)(32.2)Q[54.4fQ^2],$$

$$\text{or:}\quad fQ^3 \approx 134 \quad \text{with Q in ft}^3/\text{s}$$

$$\text{Guess}\quad f \approx 0.02,\quad Q = (134/0.02)^{1/3} \approx 18.9\ \frac{\text{ft}^3}{\text{s}},\quad \text{Re} = \frac{\rho(Q/A)D_h}{\mu} \approx 209000$$

Iterate: fbetter ≈ 0.0179, Qbetter ≈ 19.6 ft^3/s, Rebetter ≈ 216500. The process converges to

$f \approx 0.01784$, $V \approx 80.4$ ft/s, **Q ≈ 19.6 ft^3/s**. *Ans.*

Using effective diameter:

Table 6.4: 2θ=60 deg ➔ P_{0f}=53.3➔ D_{eff}=64/53.3*D_h=1.2 D_h=0.5196 and

ε/D_{eff}=0.000289

$$h_f = f\frac{L}{D_h}\frac{(Q/A)^2}{2g} = 54.4 fQ^2$$

$$Power = 1 = \rho g Q f h_f = (0.00234)(32.2)Q(54.4 fQ^2) \Rightarrow fQ^3 = 134$$

1st iteration:

Guess: f=0.02 ➔ Q=18.9 ft^3/s ➔ $\mathrm{Re} = \dfrac{\rho(Q/A)D_{eff}}{\mu} = 251000$

f=f(Re=251000, ε/D_{eff}=0.000289)➔ f=0.017207

2nd iteration:

f=0.017207 ➔ Q=19.82 ft^3/s ➔ $\mathrm{Re} = \dfrac{\rho(Q/A)D_{eff}}{\mu} = 263100$

f=f(Re=263100, ε/D_{eff}=0.000289)➔ f=0.017122

3rd iteration:

f=0.017122 ➔ Q=19.85 ft^3/s ➔ $\mathrm{Re} = \dfrac{\rho(Q/A)D_{eff}}{\mu} = 263500$

f=f(Re=263500, ε/D_{eff}=0.000289)➔ f=0.0171186 converged

➔ Q=19.855 ft^3/s

$$\frac{Q(D_{eff}) - Q(D_h)}{Q(D_h)} = \frac{19.855 - 19.6}{19.6} = 1.3\%$$

Using effective diameter gives 1. 3% lager flow rate.