6.76 The small turbine in Fig. P6.76 extracts 400 W of power from the water flow. Both pipes are wrought iron. Compute the flow rate Q m^3/h. Why are there two solutions? Which is better?

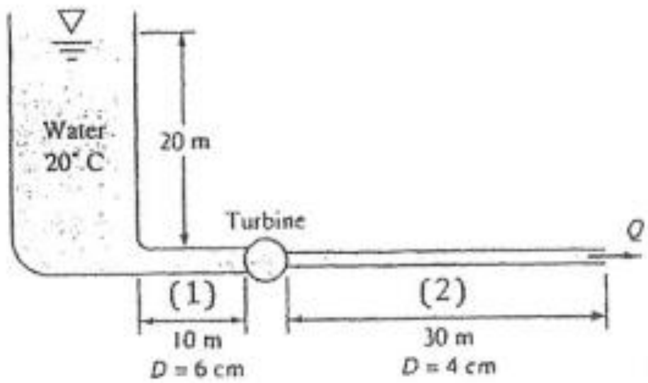
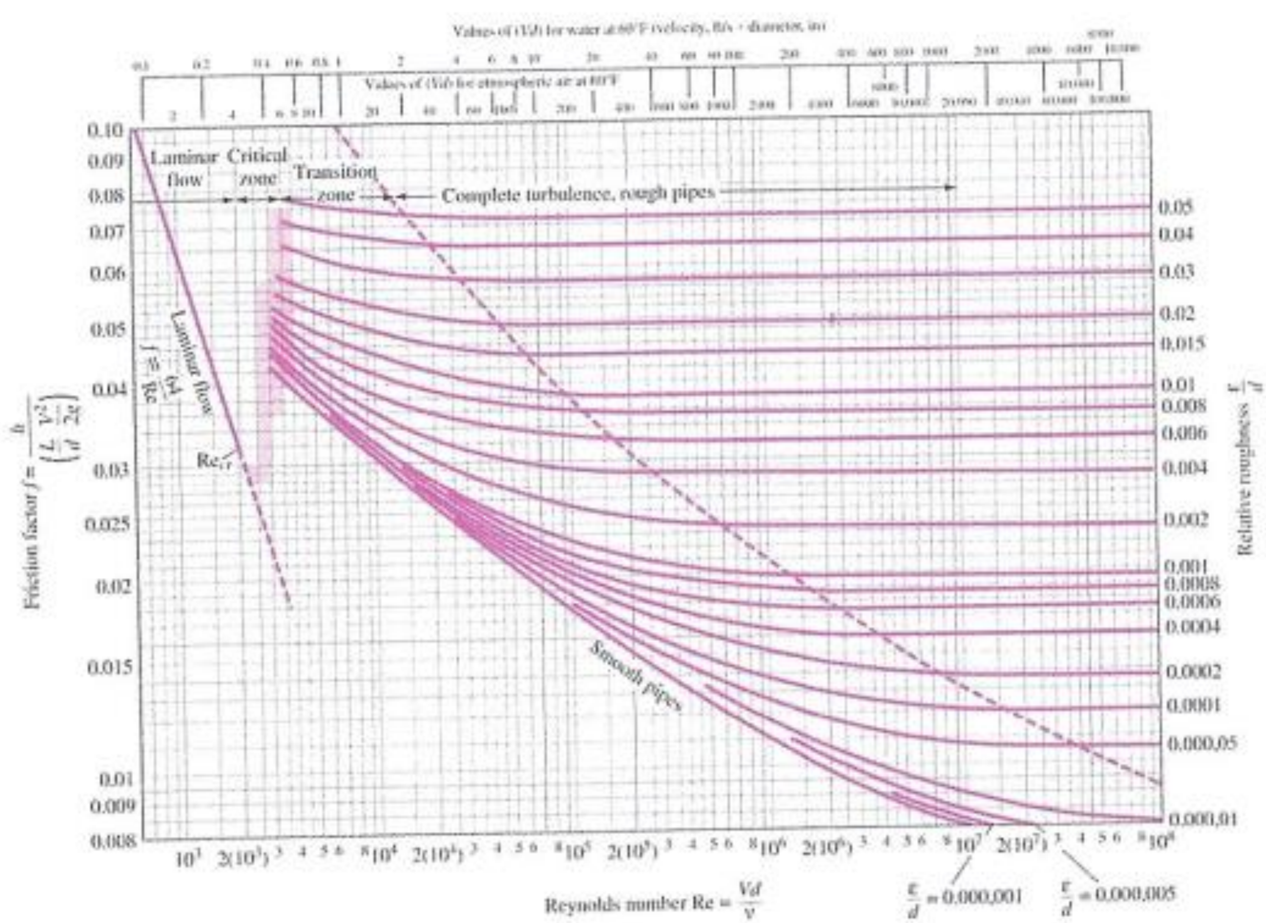


Fig. P6.76



Energy equation:

$$\left(\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1\right)_1=\left(\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2\right)_2+h_f\quad +h_T$$

$$h_f=f\frac{L}{d}\frac{V^2}{2g}$$

Assume the Velocity at the Top of the water tank is small enough to be assumed as Zero : $V_1 = 0$.

Also, the pressure at the Top of the water tank and Exit is same as atmospheric Pressure : $p_1 = p_2$

So, the Energy Equation to be $z_1 = \frac{V_2^2}{2g} + z_2 + h_{f1} + h_{f2} + h_{turbine}$

$$z_1 - z_2 = \frac{V_2^2}{2g} + h_{f1} + h_{f2} + h_{turbine}$$

Also, $h_f = f\frac{L}{d}\frac{V^2}{2g}$ and $h_{turbine} = \frac{P}{\rho g Q}$

$$\text{So, } z_1 - z_2 = \frac{V_2^2}{2g} + f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} + f_2\frac{L_2}{d_2}\frac{V_2^2}{2g} + \frac{P}{\rho g Q}$$

Furthermore, we can represent V_1, V_2 as a flow rate Q as shown in below.

$$Q = \frac{\pi}{4}d_1^2 V_1 \quad V_1 = \frac{4Q}{\pi d_1^2}$$

$$Q = \frac{\pi}{4}d_2^2 V_2 \quad V_2 = \frac{4Q}{\pi d_2^2}$$

Then, the Energy Equation can be written as shown in below.

$$(z_1 - z_2)Q = \frac{V_2^2 Q}{2g} + f_1\frac{16L_1Q^3}{2g\pi^2 d_1^5} + f_2\frac{16L_2Q^3}{2g\pi^2 d_2^5} + \frac{P}{\rho g}$$

$$(20)Q = \frac{16Q^3}{2g\pi^2 d_2^4} + f_1\frac{16L_1Q^3}{2g\pi^2 d_1^5} + f_2\frac{16L_2Q^3}{2g\pi^2 d_2^5} + \frac{P}{\rho g}$$

Then, plug in all variables as shown in below, and rearranging it

$$(20)Q = \frac{16Q^3}{2\times 9.81\pi^2 0.04^4} + f_1\frac{16\times 10\times Q^3}{2\times 9.81\pi^2 0.06^5} + f_2\frac{16\times 30\times Q^3}{2\times 9.81\pi^2 0.04^5} + \frac{400}{998\times 9.81}$$

$$\frac{400}{998\times 9.81} = 20Q - \frac{16Q^3}{2\times 9.81\pi^2 0.04^4} - f_1\frac{16\times 10\times Q^3}{2\times 9.81\pi^2 0.06^5} - f_2\frac{16\times 30\times Q^3}{2\times 9.81\pi^2 0.04^5}$$

$$\frac{400}{998\times 9.81} = 20Q - \frac{8Q^3}{9.81\pi^2 0.04^4} - f_1\frac{8\times 10\times Q^3}{9.81\pi^2 0.06^5} - f_2\frac{8\times 30\times Q^3}{9.81\pi^2 0.04^5}$$

Finally, we can get a Cubic equation as shown in below.

$$Q^3(1062588.184f_1 + 24207087.07f_2 + 32276.11609) - 20Q + 0.040856432 = 0$$

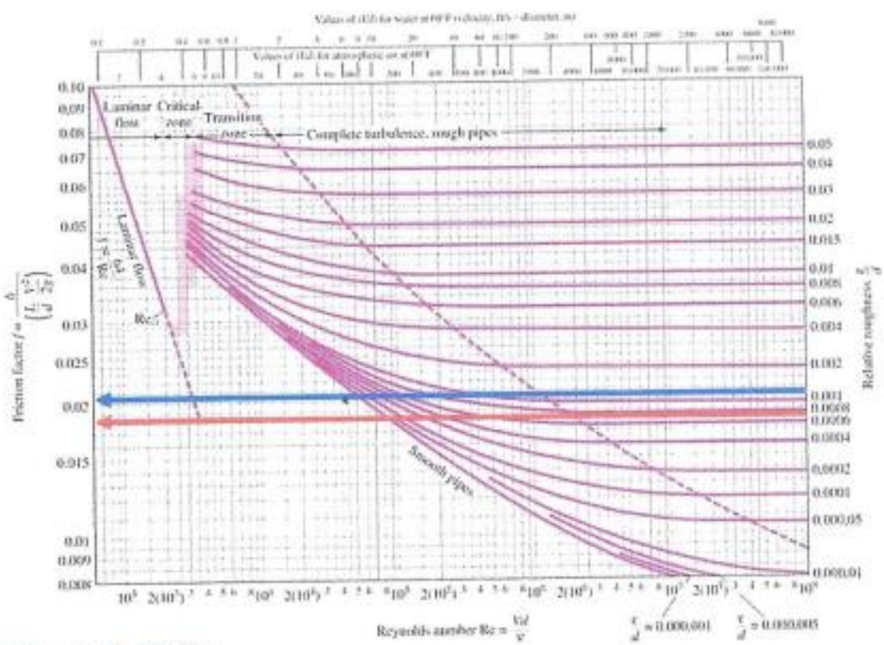
To solve this Equation, we need to assume f_1 and f_2.

To do that, let's find out $\frac{\epsilon}{d_1}, \frac{\epsilon}{d_2}$

We have $d_1 = 0.06m, \quad d_2 = 0.04m, \quad \epsilon = 0.046\text{mm} = 0.000046\text{m}$

So, $\frac{\epsilon}{d_1} = 0.000766666, \quad \frac{\epsilon}{d_2} = 0.00115$

First, assume Re⇒∞ to get f_1 and f_2. Then, at below chart, we can find the f_1 and f_2



So, f_1=0.0181 and f_2=0.0205

$$Q^3(1062588.184 \times 0.0181 + 24207087.07 \times 0.0205 + 32276.11609) - 20Q + 0.040856432 = 0$$

$$Q^3(547754.2472) - 20Q + 0.040856432 = 0$$

Use MATLAB to solve above Cubic equation as shown in below.

```
New to MATLAB? See resources for Getting Started.

>> p=[547754.2472 0 -20 0.040856432]
z = roots(p)

p =

   1.0e+05 *

   5.477542472000000                  0  -0.000200000000000   0.000000408564320

z =

  -0.006881287527623
   0.004439561588403
   0.002441320939220
```

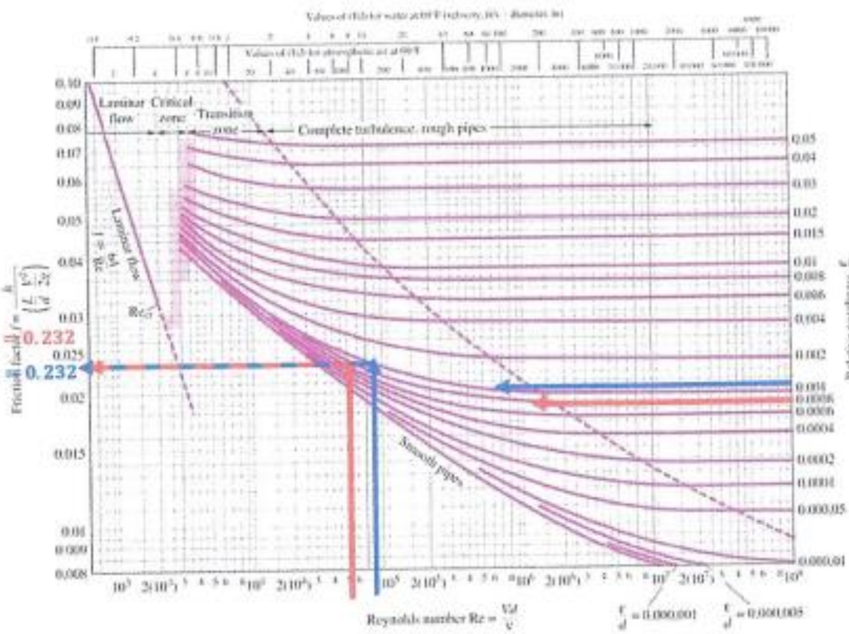
$Q_1 = -0.00688$ The flow rate can't be negative
$Q_2 = 0.00444\ m^3/s$
$Q_3 = 0.00244\ m^3/s$

[The negative (meaningless) solution is Q = - 0.0069 m³/hr.] Both solutions (a) and (b) are valid mathematically. **Solution (*b*) is preferred** – the same power for 43% less water flow, and the turbine captures 16.3 m of the available 20 m head. Solution (*a*) is also unrealistic, because a real turbine's power increases with water flow rate. Turbine (*a*) would generate more than 400 W.

Use $Q_3 = 0.00244\, m^3/s$

$$V_1 = \frac{4Q}{\pi d_1^2} = \frac{4\times0.00244}{\pi\times0.06^2} = 0.86297\text{m/s} \qquad Re_1 = \frac{\rho V d_1}{\mu} = \frac{998\times0.86297\times0.06}{0.001} = 51674.6 = 5.2\times10^4$$

$$V_2 = \frac{4Q}{\pi d_2^2} = \frac{4\times0.00244}{\pi\times0.04^2} = 1.94169\text{m/s} \qquad Re_2 = \frac{\rho V d_1}{\mu} = \frac{998\times1.94169\times0.04}{0.001} = 77512.3 = 7.8\times10^4$$



Then we can repeat previous procedure by using new f_1, f_2

So, f_1=0.0232 and f_2=0.0232

$$Q^3(1062588.184\times0.0232 + 24207087.07\times0.0232 + 32276.11609) - 20Q + 0.040856432 = 0$$

$$Q^3(618532.582) - 20Q + 0.040856432 = 0$$

Use MATLAB to solve above Cubic equation again as shown in below.

```
New to MATLAB? See resources for Getting Started.
>> p = [618532.582 0 -20 0.040856432]
r = roots(p)

p =

   1.0e+05 *

   6.185325820000001                    0  -0.000200000000000   0.000000408564320

r =

  -0.006516926028129
   0.003952617532711
   0.002564308495415
```

$Q_1 = -0.00652$ The flow rate can't be negative.
$Q_2 = 0.00395\, m^3/s \Rightarrow 14.22\, m^3/h$
$Q_3 = 0.00256\, m^3/s \Rightarrow 9.22\, m^3/h$

(a) $h_{L1} = \frac{400}{998\times9.81\times.00437} = 9.35$ m

(b) $h_{L2} = \frac{400}{998\times9.81\times.0025} = 16.3$ m