Intermediate Fluid Mechanics
Exam 1 Review

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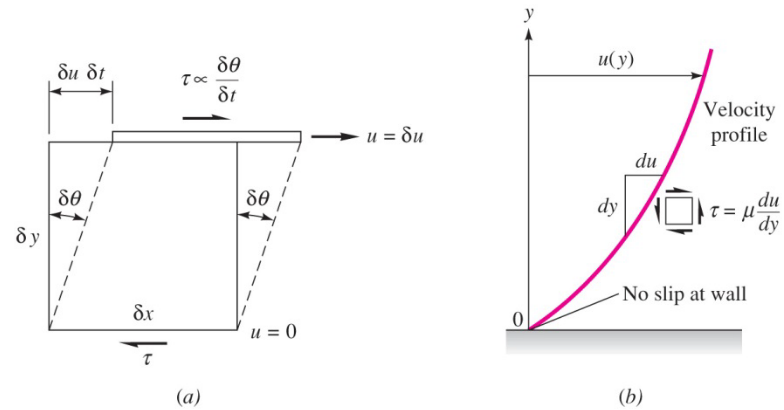
Viscosity

- Shear stress

$$\tau \propto \frac{\delta\theta}{\delta t}; \qquad \tan\delta\theta = \frac{\delta u \delta t}{\delta y}$$

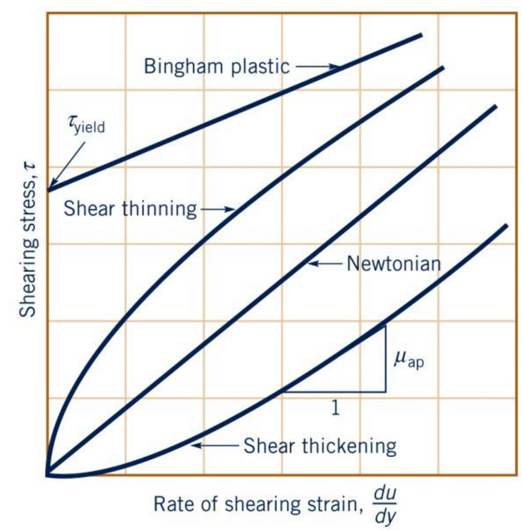
  - τ: Shear stress (N/m^2 or lbf/ft^2)
  - $\delta\theta$: Shear strain angle



- Newtonian fluid

$$\tau = \mu \frac{du}{dy}$$

  - μ: Dynamic viscosity (N·s/m^2 or lbf·s/ft^2)
  - $\nu = \mu/\rho$: Kinematic viscosity (m^2/s or ft^2/s)
  - Shear force = $\tau \cdot A$



- Non-Newtonian fluid

$$\tau \propto \left(\frac{du}{dy}\right)^n$$

Vapor Pressure and Cavitation

- **Vapor pressure**: Below which a liquid evaporates, i.e., changes to a gas. If the pressure drop is due to
  - Temperature effect: Boiling
  - Fluid velocity: **Cavitation**

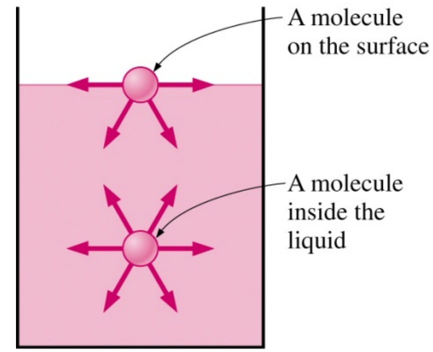
Cavitation formed on a marine propeller

Surface Tension

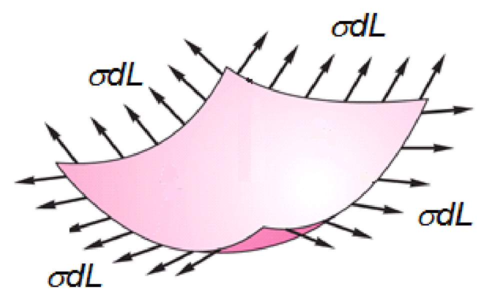
- **Surface tension force**: The force developed at the interface of two immiscible fluids (e.g., liquid-gas) due to the unbalanced molecular cohesive forces at the fluid surface.

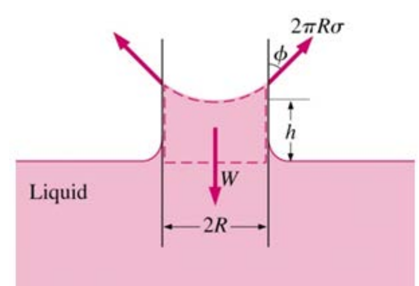
$$F_\sigma = \sigma \cdot L$$

- F_σ = Line force with direction normal to the cut
- σ = Surface tension [N/m], the intensity of the molecular attraction per unit length
- L = Length of cut through the interface



Attractive forces acting on a liquid molecule at the surface and deep inside the liquid





The forces acting on a liquid column that has risen in a tube due to the capillary effect

Equations of Fluid Motions

- Newton's 2nd law (per unit volume):

$$\rho \underline{a} = \sum \underline{f}$$

where, $\sum \underline{f}$ = $\underline{f}_{\text{body}}$ + $\underline{f}_{\text{surface}}$ and $\underline{f}_{\text{surface}} = \underline{f}_{\text{pressure}} + \underline{\text{f}}_{\text{shear}}$

- Viscous fluids flow (Navier-Stokes equation):

$$\rho \underline{a} = -\rho \mathrm{g} \hat{\boldsymbol{k}} - \nabla p + \mu \nabla^2 \underline{V}$$

- Inviscid fluids flow (μ = 0; Euler equation):

$$\rho \underline{a} = -\rho \mathrm{g} \hat{\boldsymbol{k}} - \nabla p$$

- Fluids at rest (No motion, i.e.,$\underline{a}$ = 0):

$$\nabla p = -\rho \mathrm{g} \hat{\boldsymbol{k}}$$

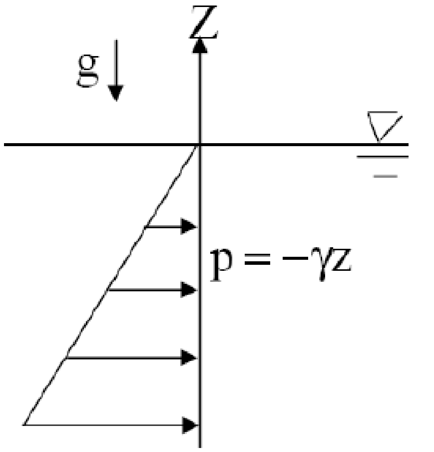
Pressure Variation with Elevation

For fluids at rest,

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

and

$$\frac{\partial p}{\partial z} = -\gamma$$



For constant γ (e.g., liquids), by integrating the above equations,

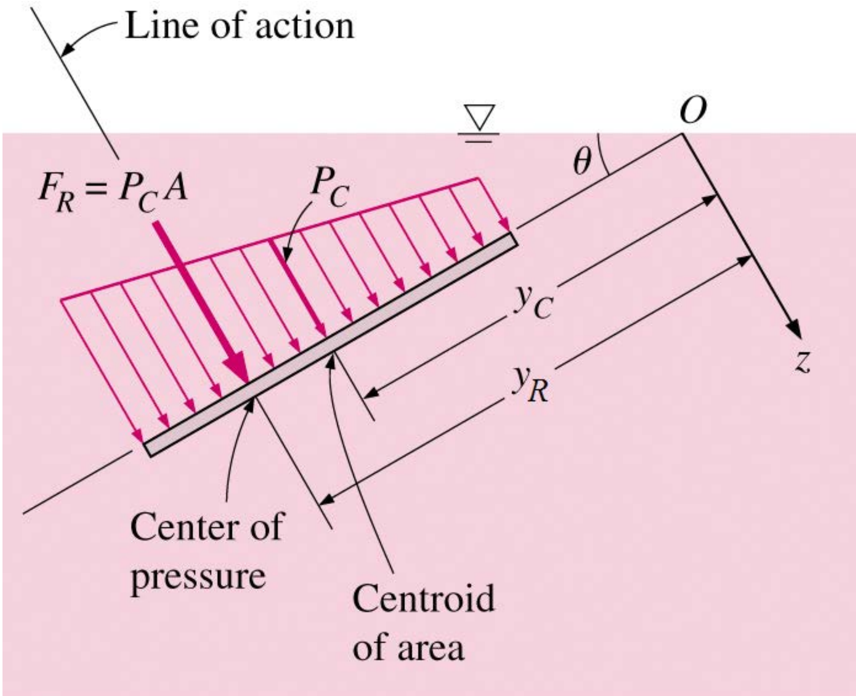
$$p = -\gamma z + C$$

At z = 0, $p = C = 0$ (gage),

$$\therefore p = -\gamma z$$

$\Rightarrow$ The pressure increases linearly with depth.

Hydrostatic Forces: (1) Inclined surfaces



- Average pressure on the surface

$$\bar{p} = p_C = \gamma h_c$$

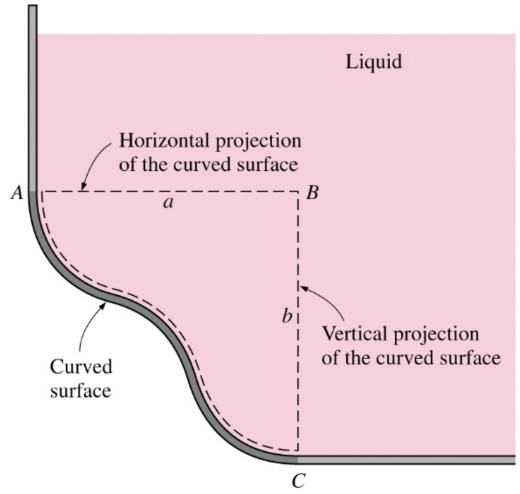
- The magnitude of the resultant force is simply

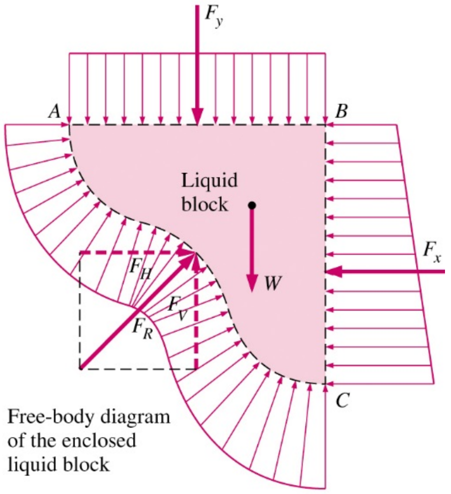
$$F_R = \bar{p}A = \gamma h_c A$$

- Pressure center

$$y_R = y_c + \frac{I_{xc}}{y_c A}$$

Hydrostatic Forces: (2) Curved surfaces





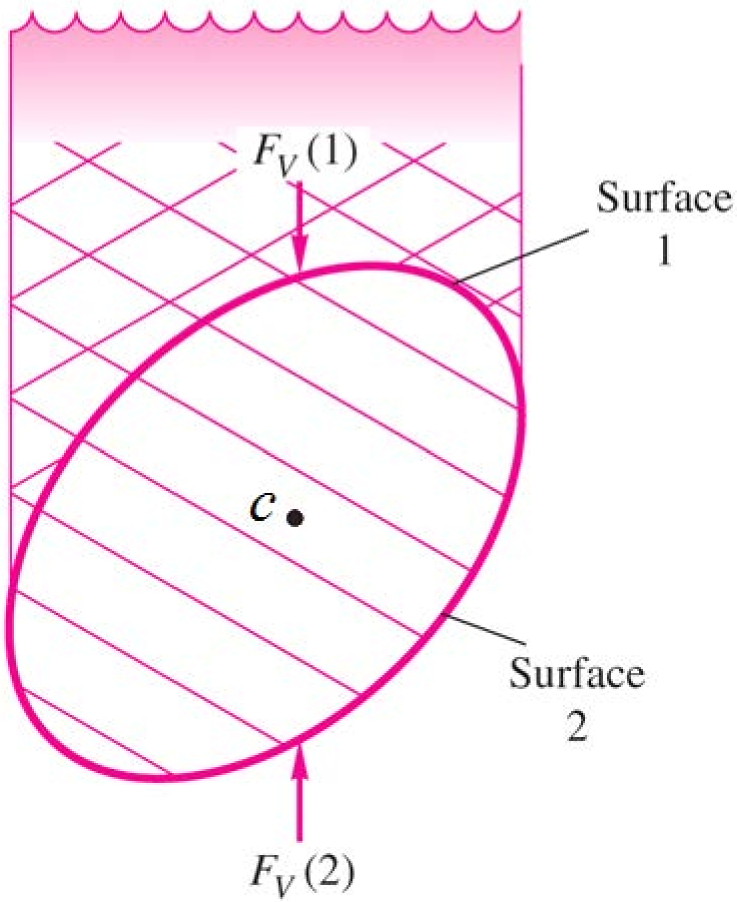
$$F_x = \bar{p}_{\text{proj}} \cdot A_{\text{proj}}$$

$$F_y = \gamma V\!\!\!/_{\text{above } AB}$$

$$W = \gamma V\!\!\!/_{ABC}$$

- Horizontal force component: $F_H = F_x$
- Vertical force component: $F_V = F_y + W = \gamma V\!\!\!/_{\text{total volume above } AC}$
- Resultant force: $F_R = \sqrt{F_H^2 + F_V^2}$

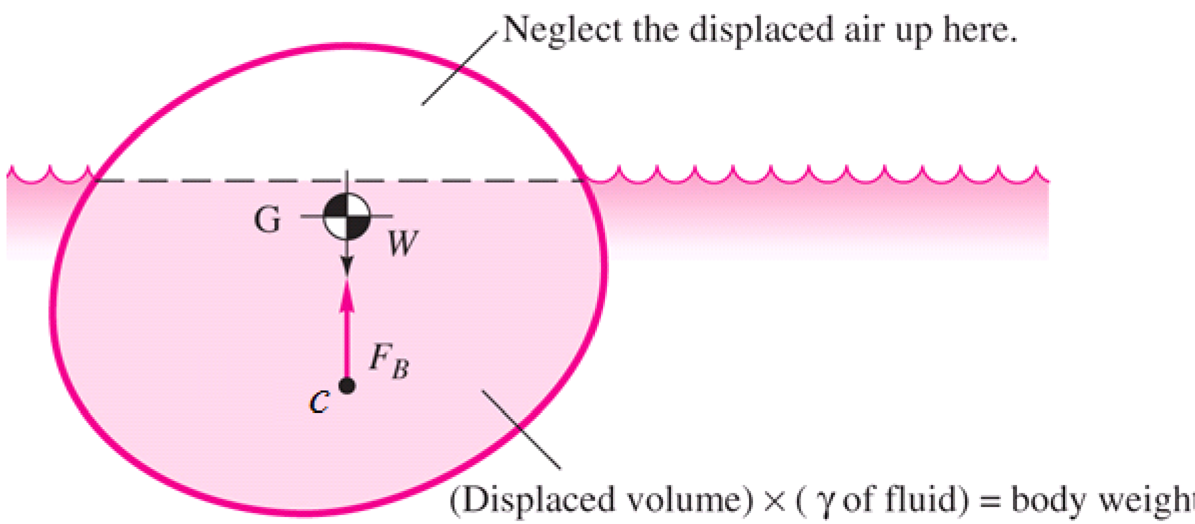
Buoyancy: (1) Immersed bodies



$$F_B = F_{V2} - F_{V1} = \gamma V\!\!\!/$$

- Fluid weight equivalent to body volume $V\!\!\!/$
- Line of action (or the center of buoyancy) is through the centroid of $V\!\!\!/$, c

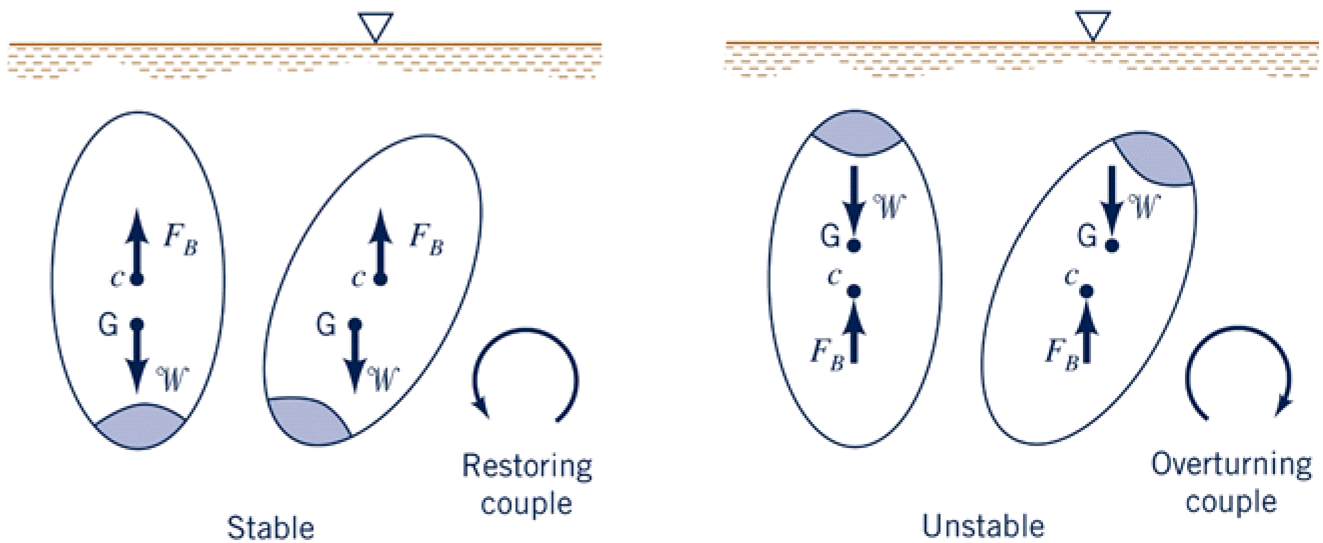
Buoyancy: (2) Floating bodies



$F_B = \gamma \forall_{\text{displaced volume}}$ (i.e., the weight of displaced water)

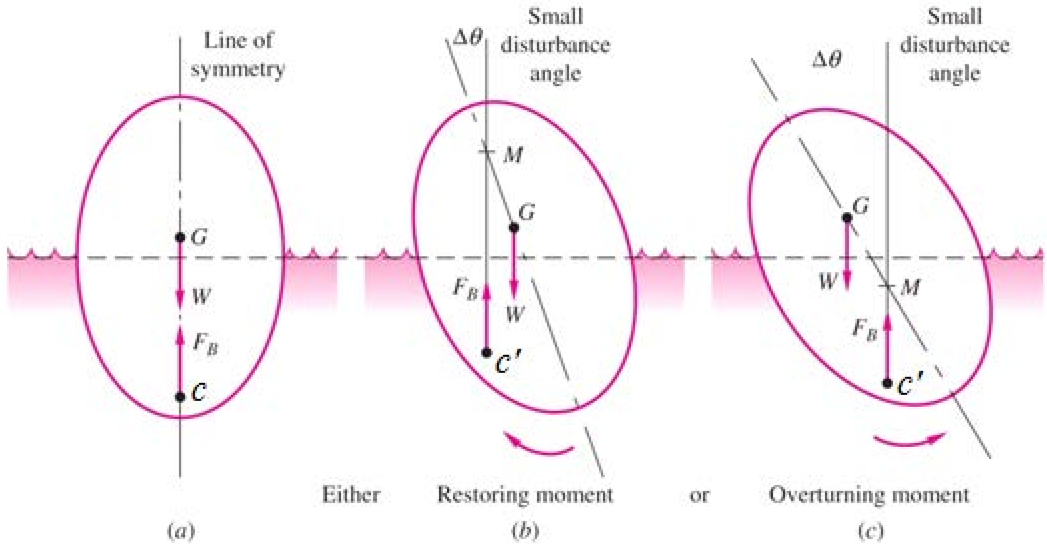
Line of action (or the center of buoyancy) is through the centroid of the displaced volume

Stability: (1) Immersed bodies



- If c is above G: Stable (righting moment when heeled)
- If c is below G: Unstable (heeling moment when heeled)

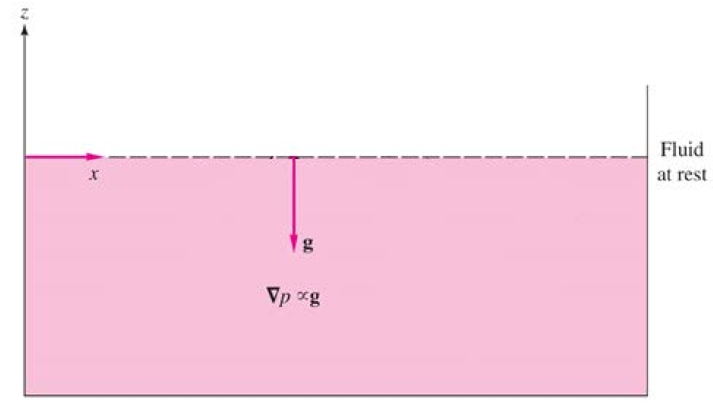
Stability: (2) Floating bodies

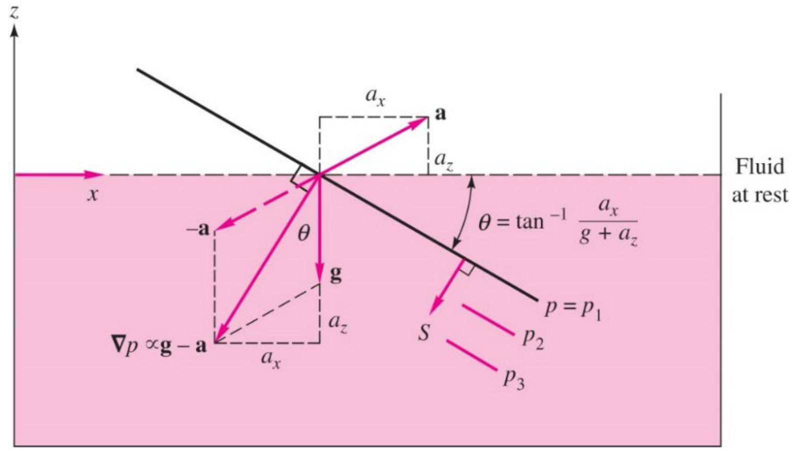


- $GM > 0$: Stable (M is above G)
- $GM < 0$: Unstable (G is above M)

$$GM = \frac{I_{00}}{V\!\!\!/} - CG$$

Rigid-body motion: (1) Translation





- Fluid at rest
  - o $\frac{\partial p}{\partial z} = -\rho g$
  - o $p = \rho g z$

- Rigid-body in translation with a constant acceleration,

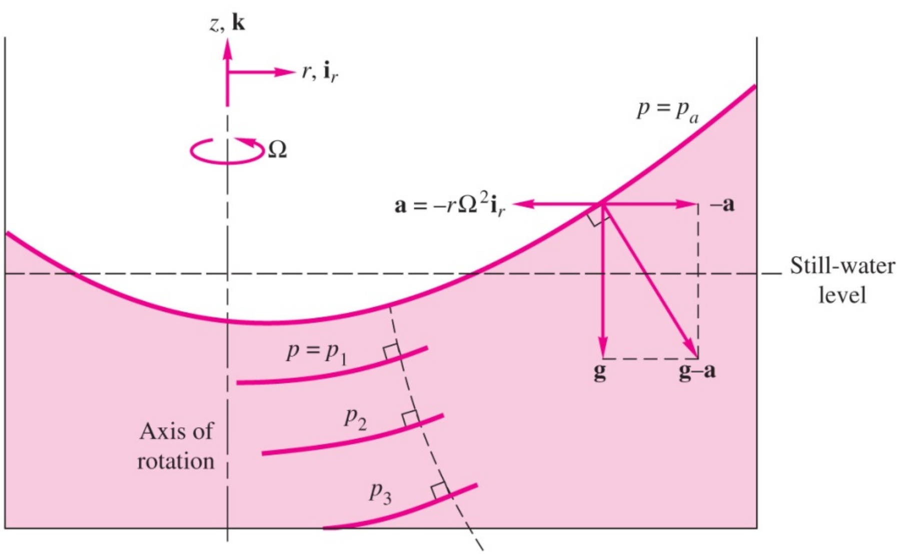
$$\underline{a} = a_x \hat{\boldsymbol{\imath}} + a_z \widehat{\boldsymbol{k}}$$

  - o $\frac{\partial p}{\partial s} = -\rho G$
  - o $p = \rho G s$

$$G = (a_x^2 + (g + a_z)^2)^{\frac{1}{2}}$$

$$\theta = \tan^{-1} \frac{a_x}{g + a_z}$$

Rigid-body motion: (2) Rotation



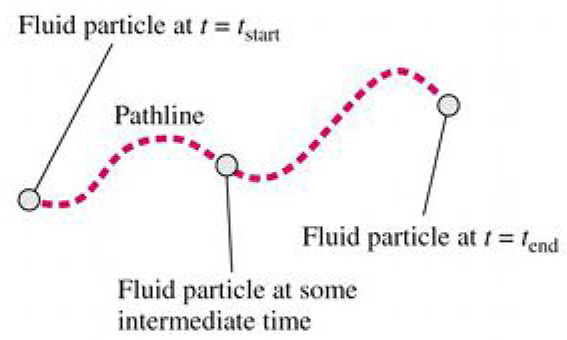
- Rigid-body in translation with a constant rotational speed Ω,

$$\underline{a} = -r\Omega^2 \hat{\boldsymbol{e}}_{\boldsymbol{r}}$$

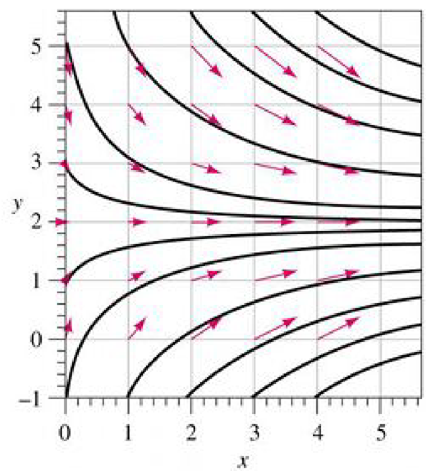
  - $\frac{\partial p}{\partial r} = \rho r \Omega^2$ and $\frac{\partial p}{\partial z} = -\rho g$
  - $p = \frac{\rho}{2} r^2 \Omega^2 - \rho g z + C$
  - $z = \frac{p_0 - p}{\rho g} + \frac{\Omega^2}{2g} r^2$

Flow Patterns

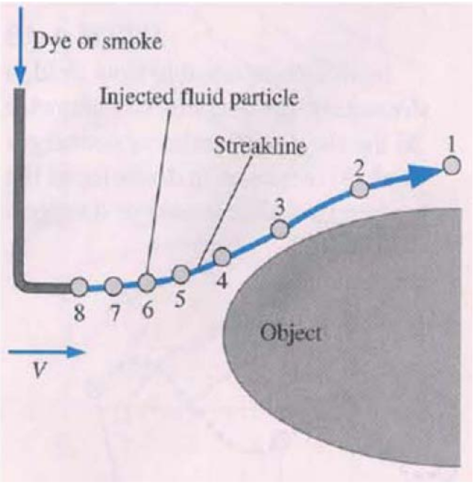
- **Pathline**: The actual path traveled by a given fluid particle.
- **Streamline**: A line that is everywhere tangent to the velocity vector at a given instant.
- **Streakline**: The locus of particles which have earlier passed through a particular point.
- For steady flow, all three lines coincide.



Pathline



Streamline



Streakline

Bernoulli Equation

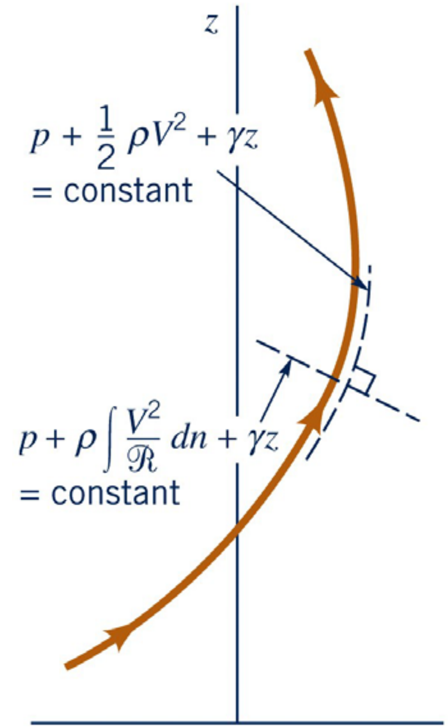
Integration of the Euler equation for a **steady incompressible** flow:

- Along a streamline:

$$p + \frac{1}{2}\rho V^2 + \gamma z = \text{Constant}$$

- Across the streamline:

$$p + \rho \int \frac{V^2}{\Re} dn + \gamma z = \text{Constant}$$



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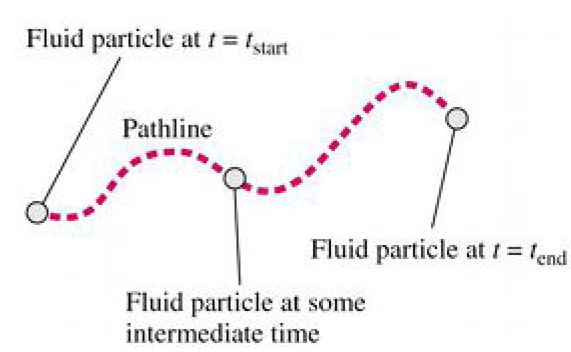
Flow Kinematics: (1) Lagrangian Description



- Keep track of individual fluid particles

$$\underline{V_p}(t) = \frac{d\underline{x}}{dt} = u_p(t)\hat{\boldsymbol{\imath}} + v_p(t)\hat{\boldsymbol{\jmath}} + w_p(t)\hat{\boldsymbol{k}}$$

$$u_p = \frac{dx}{dt}, v_p = \frac{dy}{dt}, w_p = \frac{dz}{dt}$$

$$\underline{a_p} = \frac{d\underline{V_p}}{dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$a_x = \frac{du_p}{dt}, a_y = \frac{dv_p}{dt}, a_z = \frac{dw_p}{dt}$$

Flow Kinematics: (2) Eulerian Description

- Focus attention on a fixed point in space

$$\underline{V}(\underline{x},t) = u(\underline{x},t)\hat{\boldsymbol{\imath}} + v(\underline{x},t)\hat{\boldsymbol{\jmath}} + w(\underline{x},t)\hat{\boldsymbol{k}}$$

$$\underline{a} = \frac{D\underline{V}}{Dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

Or,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Acceleration and material derivatives –Contd.

- Acceleration

$$\underline{a} = \frac{D\underline{V}}{Dt} = \underbrace{\frac{\partial \underline{V}}{\partial t}}_{\substack{\text{Local} \\ \text{acc.}}} + \underbrace{\left(\underline{V} \cdot \nabla\right)\underline{V}}_{\substack{\text{Convective} \\ \text{acc.}}}$$

  - $\frac{\partial \underline{V}}{\partial t}$ = Local or temporal acceleration. Velocity changes with respect to time at a given point
  - $\left(\underline{V} \cdot \nabla\right)\underline{V}$ = Convective acceleration. Spatial gradients of velocity

- Material derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \left(\underline{V} \cdot \nabla\right)$$

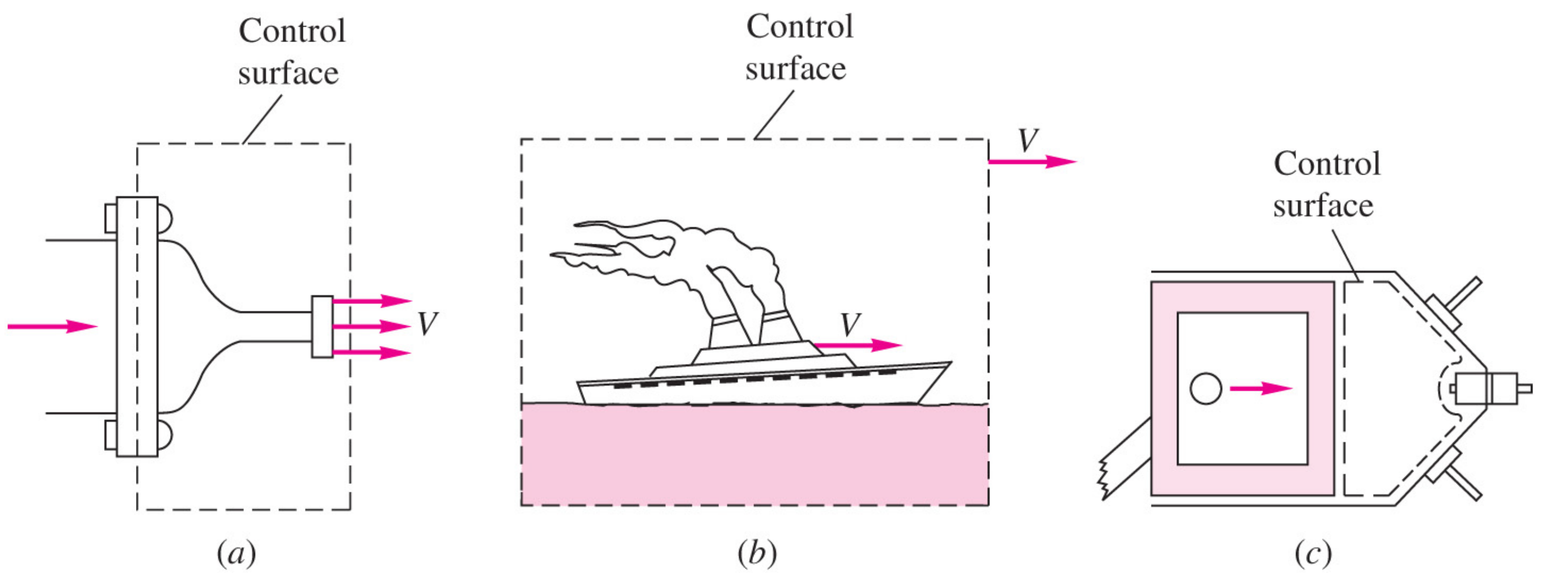
where

$$\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$$

System vs. Control volume

- **System**: A collection of real matter of fixed identity.

- **Control volume (CV)**: A geometric or an imaginary volume in space through which fluid may flow. A CV may move or deform.



Reynolds Transport Theorem (RTT)

- In fluid mechanics, we are usually interested in a region of space, i.e., CV and not particular systems. Therefore, we need to transform GDE's from a system to a CV, which is accomplished through the use of RTT

$$\underbrace{\frac{DB_{\text{sys}}}{Dt}}_{\substack{\text{time rate of change}\\ \text{of } B \text{ for a system}}} = \underbrace{\frac{D}{Dt}\int_{\text{CV}(\underline{x},t)} \beta\rho dV}_{\substack{\text{time rate of change}\\ \text{of } B \text{ in CV}}} + \underbrace{\int_{\text{CS}(\underline{x},t)} \beta\rho \underline{V}_R \cdot d\underline{A}}_{\substack{\text{net flux of } B\\ \text{across CS}}}$$

where, $\beta = \frac{dB}{dm} = \left(1, \underline{V}, e\right)$ for $B = (m, m\underline{V}, E)$

- Fixed CV,

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{CV}} \beta\rho dV + \int_{\text{CS}} \beta\rho \underline{V} \cdot d\underline{A}$$

Note:

$$B_{\text{CV}} = \int_{\text{CV}} \beta dm = \int_{\text{CV}} \beta\rho dV$$

$$\dot{B}_{\text{CS}} = \int_{\text{CS}} \beta d\dot{m} = \int_{\text{CS}} \beta\rho \underline{V} \cdot d\underline{A}$$

Continuity Equation

- RTT with $B = m$ and $\beta = 1$,

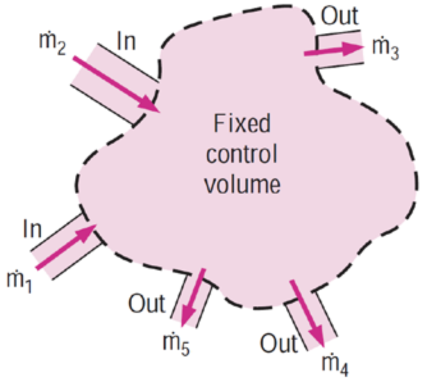
$$\frac{\partial}{\partial t}\int_{\mathrm{CV}} \rho dV + \int_{\mathrm{CS}} \rho \underline{V_r} \cdot \underline{n}\, dA = 0$$

- Steady flow with fixed CV,

$$\int_{\mathrm{CS}} \rho \underline{V} \cdot d\underline{A} = 0$$

- One-dimensional

$$\sum \dot{m}_{\mathrm{out}} - \sum \dot{m}_{\mathrm{in}} = 0$$



Note: $\dot{m} = \rho Q = \rho V A$

Linear Momentum Equation

- RTT with $B = m\underline{V}$ and $\beta = \underline{V}$,

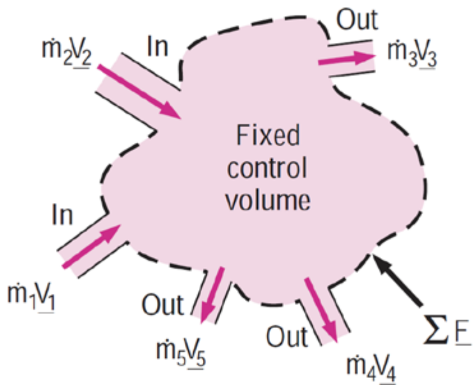
$$\frac{\partial}{\partial t}\left(\int_{\mathrm{CV}} \underline{V}\rho dV\right) + \int_{\mathrm{CS}} \underline{V}\rho(\underline{V_r}\cdot\underline{n})\, dA = \sum \underline{F}$$

- Steady flow with fixed CV,

$$\int_{\mathrm{CS}} \underline{V}\rho(\underline{V}\cdot\underline{n})\, dA = \sum \underline{F}$$

- One-dimensional,

$$\sum(\dot{m}\underline{V})_{\mathrm{out}} - \sum(\dot{m}\underline{V})_{\mathrm{in}} = \sum \underline{F}$$



or in component forms,

$$\begin{aligned}
\sum(\dot{m}u)_{\mathrm{out}} - \sum(\dot{m}u)_{\mathrm{in}} &= \sum F_x \\
\sum(\dot{m}v)_{\mathrm{out}} - \sum(\dot{m}v)_{\mathrm{in}} &= \sum F_y \\
\sum(\dot{m}w)_{\mathrm{out}} - \sum(\dot{m}w)_{\mathrm{in}} &= \sum F_z
\end{aligned}$$

Note: If $\underline{V} = u\hat{\boldsymbol{\imath}} + v\hat{\boldsymbol{\jmath}} + w\hat{\boldsymbol{k}}$ is normal to CS, $\dot{m} = \rho V A$, where $V = |\underline{V}|$.

Linear Momentum Equation – Cont.

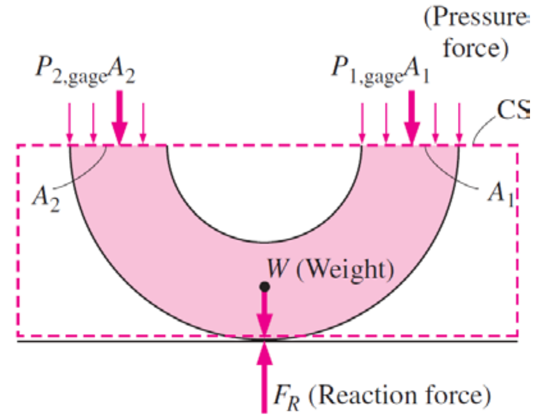
- External forces:

$$\sum \underline{F} = \sum \underline{F}_{\text{body}} + \sum \underline{F}_{\text{surface}} + \sum \underline{F}_{\text{other}}$$

  - $\sum \underline{F}_{\text{body}} = \sum \underline{F}_{\text{gravity}}$
    - $\sum \underline{F}_{\text{gravity}}$: gravity force (i.e., weight)
  - $\sum \underline{F}_{\text{Surface}} = \sum \underline{F}_{\text{pressure}} + \sum \underline{F}_{\text{friction}}$
    - $\sum \underline{F}_{\text{pressure}}$: pressure forces normal to CS

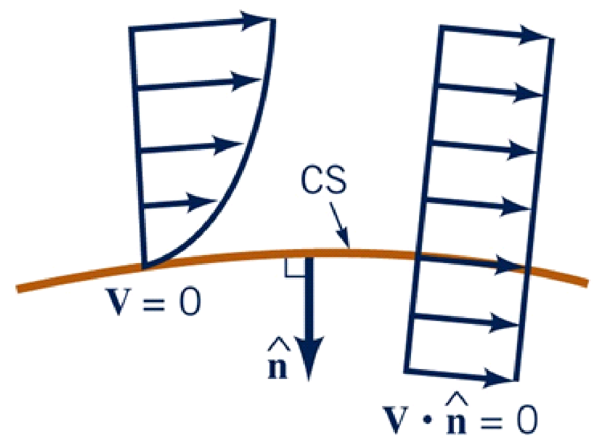
$$\underline{F}_{pressure} = \int_{cs} p_{gage}(-\underline{n})dA$$

    - $\sum \underline{F}_{\text{friction}}$: viscous friction forces tangent to CS
  - $\sum \underline{F}_{\text{other}}$: anchoring forces or reaction forces



An 180° elbow supported by the ground

In most flow systems, the force $\vec{F}$ consists of weights, pressure forces, and reaction forces. Gage pressures are used here since atmospheric pressure cancels out on all sides of the control surface.



Note: Shearing forces can be avoided by carefully selecting the CV such that CS's are parallel with the flow direction.

Angular Momentum Equation

- RTT with $B = \int \underline{r} \times \underline{V} dm$ and $\beta = \underline{r} \times \underline{V}$,

$$\sum \underline{M_0} = \frac{\partial}{\partial t}\left[\int_{CV} (\underline{r} \times \underline{V}) \rho dV\right] + \int_{CS} (\underline{r} \times \underline{V}) \rho \left(\underline{V_r} \cdot \underline{n}\right) dA$$

- Steady flow with fixed CV,

$$\sum \underline{M_0} = \int_{CS} (\underline{r} \times \underline{V}) \rho (\underline{V} \cdot \underline{n}) dA$$

- One-dimensional,

$$\sum \underline{M_0} = \sum (\underline{r} \times \underline{V})_{out} \dot{m}_{out} - \sum (\underline{r} \times \underline{V})_{in} \dot{m}_{in}$$

Energy Equation

- RTT with $B = E$ and $\beta = e$,

$$\frac{\partial}{\partial t}\int_{\mathrm{CV}} e\rho dV + \int_{\mathrm{CS}} e\rho \underline{V}\cdot d\underline{A} = \dot{Q} - \dot{W}$$

- Simplified form:

$$\frac{p_{\mathrm{in}}}{\gamma} + \alpha_{\mathrm{in}}\frac{V_{\mathrm{in}}^2}{2\mathrm{g}} + z_{\mathrm{in}} + h_p = \frac{p_{\mathrm{out}}}{\gamma} + \alpha_{\mathrm{out}}\frac{V_{\mathrm{out}}^2}{2\mathrm{g}} + z_{\mathrm{out}} + h_t + h_L$$

- V in energy equation refers to average velocity $\bar{V}$

- α : kinetic energy correction factor $= \begin{cases} 1 \text{ for uniform flow across CS} \\ 2 \text{ for laminar pipe flow} \\ \approx 1 \text{ for turbulent pipe flow} \end{cases}$

Energy Equation - Contd.

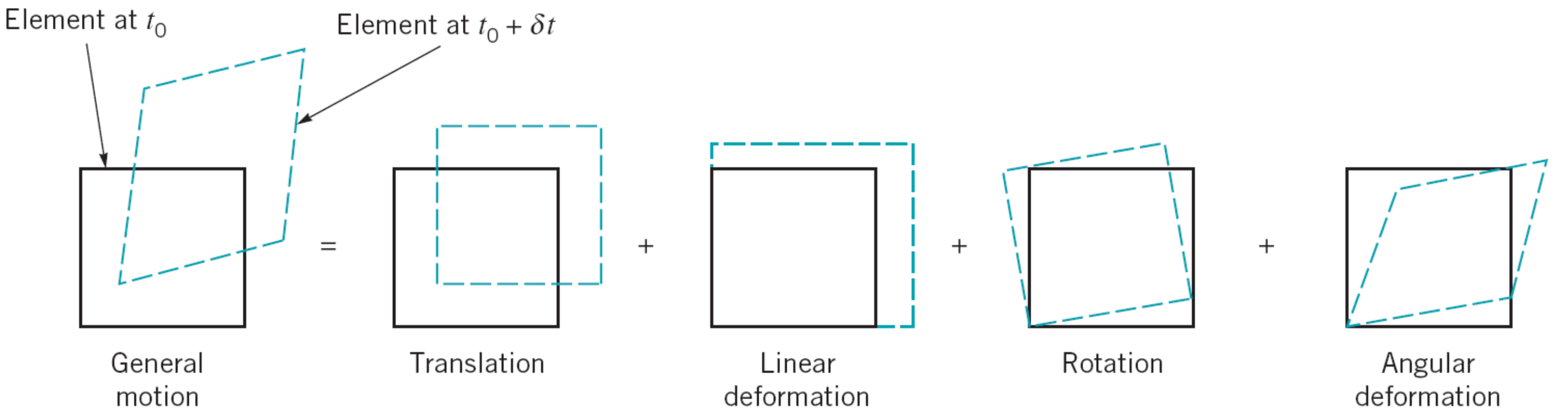
Uniform flow across CS's:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2\text{g}} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2\text{g}} + z_1 + h_t + h_L$$

- Pump head $h_p = \frac{\dot{W}_p}{\dot{m}\text{g}} = \frac{\dot{W}_p}{\rho Q\text{g}} = \frac{\dot{W}_p}{\gamma Q} \Rightarrow \dot{W}_p = \dot{m}\text{g}h_p = \rho\text{g}Qh_p = \gamma Q h_p$
- Turbine head $h_t = \frac{\dot{W}_t}{\dot{m}\text{g}} = \frac{\dot{W}_t}{\rho Q\text{g}} = \frac{\dot{W}_t}{\gamma Q} \Rightarrow \dot{W}_t = \dot{m}\text{g}h_t = \rho\text{g}Qh_t = \gamma Q h_t$
- Head loss $h_L = \text{loss}/\text{g} = (\hat{u}_2 - \hat{u}_1)/\text{g} - \dot{Q}/\dot{m}\text{g} > 0$

Differential Analysis

- Fluid Element Kinematics



- Linear deformation(dilatation): $\nabla \cdot \underline{V}$

  $\Rightarrow$ if the fluid is **incompressible** $\boldsymbol{\nabla} \cdot \underline{\boldsymbol{V}} = \mathbf{0}$

- Rotation(vorticity): $\underline{\xi} = 2\underline{\omega} = \nabla \times \underline{V}$

  $\Rightarrow$ if the fluid is **irrotational** $\boldsymbol{\nabla} \times \underline{\boldsymbol{V}} = \mathbf{0}$

- Angular deformation is related to shearing stress

  (e.g., $\tau_{ij} = 2\mu\varepsilon_{ij}$ for Newtonian fluids)

Differential Analysis
- Mass Conservation

For a fluid particle,

$$\lim_{\mathrm{CV}\to 0}\left[\int_{\mathrm{CV}}\frac{\partial \rho}{\partial t}\,d\forall + \int_{CS}\rho\underline{V}\cdot d\underline{A}\right]$$

$$= \lim_{\mathrm{CV}\to 0}\int_{\mathrm{CV}}\left[\frac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho\underline{V}\right)\right]d\forall = 0$$

$$\therefore \frac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho\underline{V}\right) = 0$$

For an incompressible flow: $\nabla\cdot\underline{V} = 0$

Differential Analysis
- Momentum Conservation

$$\lim_{\mathrm{CV}\to 0}\left[\int_{\mathrm{CV}} \frac{\partial \underline{V}}{\partial t}\rho\, d\forall + \int_{\mathrm{CS}} \underline{V}\rho\underline{V}\cdot \underline{dA}\right] = \sum \underline{F}$$

or

$$\lim_{\mathrm{CV}\to 0}\int_{\mathrm{CV}} \rho\left(\frac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\nabla\underline{V}\right) d\forall = \sum \underline{F}$$

$$\therefore \rho\left(\frac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\nabla\underline{V}\right) = \sum \underline{f} \qquad (\underline{f} = \underline{F} \text{ per unit volume})$$

$$\Rightarrow \underbrace{\rho\left(\frac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\nabla\underline{V}\right)}_{=\frac{D\underline{V}}{Dt}=\underline{a}} = \underbrace{-\rho g\hat{k}}_{\text{body force due to gravity force}} \underbrace{\underbrace{-\nabla p}_{\text{pressure force}} + \underbrace{\nabla\cdot\tau_{ij}}_{\text{viscous shear force}}}_{\text{surface force}}$$

Navier-Stokes Equations

For incompressible, Newtonian fluids,

- Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- Momentum:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho \mathrm{g}_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho \mathrm{g}_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho \mathrm{g}_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Solving the N.S. Equations

1) Set up the problem and geometry (e.g., sketches), identifying all relevant dimensions and parameters.

2) List all appropriate assumptions, approximations, simplifications, and boundary conditions.

3) Simplify the differential equations of motion (continuity and Navier-Stokes) as much as possible.

4) Integrate the equations, leading to one or more constants of integration

5) Apply boundary conditions to solve for the constants of integration.

6) Verify your results.

Exact Solutions of NS Eqns.

The flow of interest is assumed additionally (than incompressible & Newtonian), for example,

1) Steady (i.e., $\boldsymbol{\partial/\partial t = 0}$ for any variable)
2) Parallel such that the y-component of velocity is zero (i.e., $\boldsymbol{v = 0}$)
3) Purely two dimensional (i.e., $\boldsymbol{w = 0}$ and $\boldsymbol{\partial/\partial z = 0}$ for any velocity component)

e.g.)

$$\frac{\partial u}{\partial x} + \cancel{\frac{\overbrace{\partial v}^{2)}}{\partial y}} + \cancel{\frac{\overbrace{\partial w}^{3)}}{\partial z}} = 0$$

$$\rho\left[\cancel{\frac{\overbrace{\partial u}^{1)}}{\partial t}} + \cancel{u\frac{\overbrace{\partial u}^{\text{continuity}}}{\partial x}} + \cancel{\overbrace{v}^{2)}\frac{\partial u}{\partial y}} + \cancel{w\overbrace{\frac{\partial u}{\partial z}}^{3)}}\right] = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left[\cancel{\frac{\overbrace{\partial^2 u}^{\text{continuity}}}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\overbrace{\partial^2 u}^{3)}}{\partial z^2}}\right]$$

or

$$\therefore \mu\frac{d^2 u}{dy^2} = \frac{\partial p}{\partial x} - \rho g_x$$

Boundary Conditions

Common BC's:

- **No-slip condition** ($\underline{V}_{\text{fluid}} = \underline{V}_{\text{wall}}$; for a stationary wall $\underline{V}_{\text{fluid}} = 0$)
- Interface boundary condition ($\underline{V}_A = \underline{V}_B$ and $\tau_{s,A} = \tau_{s,B}$)
- Free-surface boundary condition ($p_{\text{liquid}} = p_{\text{gas}}$ and $\tau_{s,\text{liquid}} = 0$)
- Symmetry boundary condition

Other BC's:

- Inlet/outlet boundary condition
- Initial condition (for unsteady flow problem)

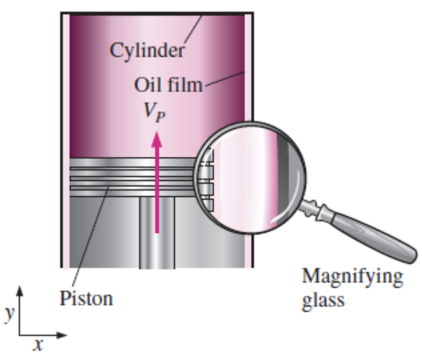


FIGURE 9–51
A piston moving at speed V_P in a cylinder. A thin film of oil is sheared between the piston and the cylinder; a magnified view of the oil film is shown. The *no-slip boundary condition* requires that the velocity of fluid adjacent to a wall equal that of the wall.

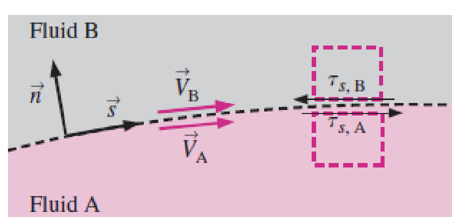


FIGURE 9–52
At an interface between two fluids, the velocity of the two fluids must be equal. In addition, the shear stress parallel to the interface must be the same in both fluids.

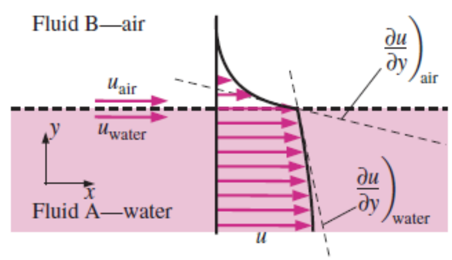


FIGURE 9–53
Along a horizontal *free surface* of water and air, the water and air velocities must be equal and the shear stresses must match. However, since $\mu_{\text{air}} \ll \mu_{\text{water}}$, a good approximation is that the shear stress at the water surface is negligibly small.

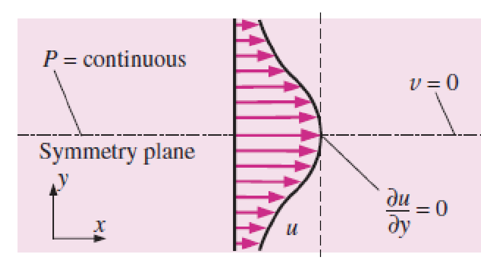


FIGURE 9–54
Boundary conditions along a plane of symmetry are defined so as to ensure that the flow field on one side of the symmetry plane is a *mirror image* of that on the other side, as shown here for a horizontal symmetry plane.

Buckingham Pi Theorem

1. List all the variable that are involved in the problem

$$u_1 = f(u_2, u_3, \cdots, u_n)$$

2. Express each of the variables in terms of basic dimensions (FLT or MLT)

3. Determine the required number of pi terms

$$\boldsymbol{r = n - m}$$

4. Select a number of repeating variables, where the number required is equal to the reference dimensions

Buckingham Pi Theorem - Cont.

5. Form a pi term by multiplying one of the nonrepeating variables by the product of the repeating variables, each raised to an exponent that will make combination dimensionless

$$u_i u_2^a u_3^b u_4^c \doteq F^0 L^0 T^0$$

6. Repeat Step 5 for each of the remaining nonrepeating variables

7. Check all the resulting pi terms to make sure they are dimensionless

8. Express the final form as a relationship along pi terms, and think about what it means

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \cdots, \Pi_r)$$