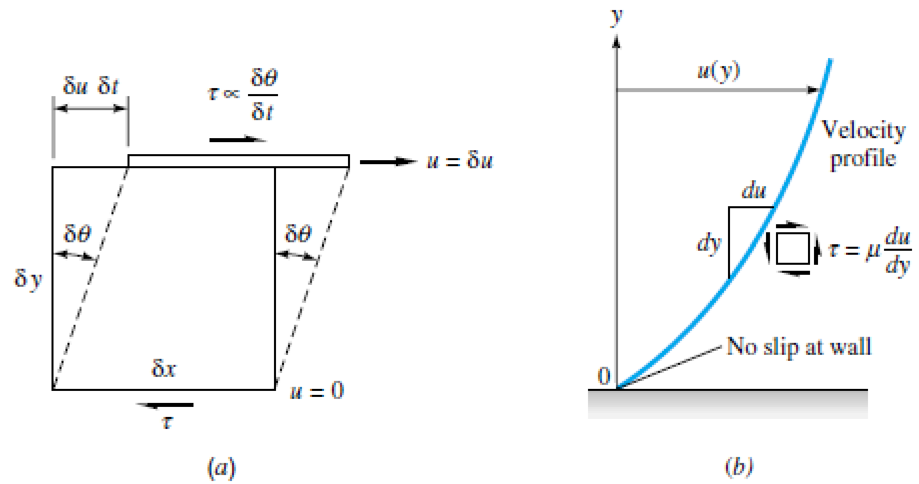
Chapter 1

$$\gamma = \rho g$$

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$

Fig. 1.4 Shear stress causes continuous shear deformation in a fluid: (*a*) a fluid element straining at a rate $\delta\theta/\delta t$; (*b*) newtonian shear distribution in a shear layer near a wall.



$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

$$\nu = \frac{\mu}{\rho}$$

$$\Delta p = \Upsilon(R_1^{-1} + R_2^{-1})$$

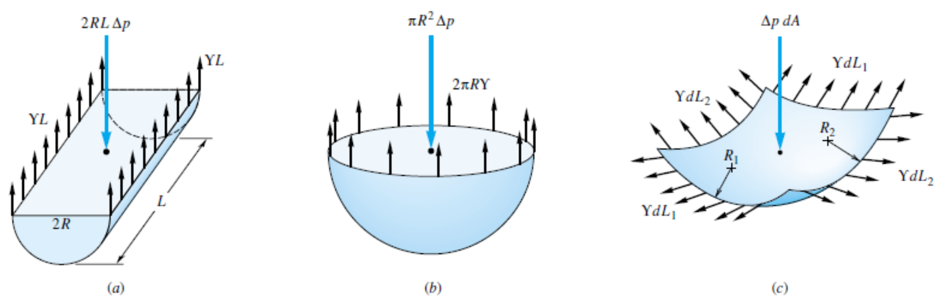


Fig. 1.9 Pressure change across a curved interface due to surface tension: (*a*) interior of a liquid cylinder; (*b*) interior of a spherical droplet; (*c*) general curved interface.

Chapter 1

1.8 Basic Flow-Analysis Techniques

There are three basic ways to attack a fluid-flow problem. They are equally important for a student learning the subject, and this book tries to give adequate coverage to each method:

1. Control-volume, or *integral* analysis (Chap. 3)
2. Infinitesimal system, or *differential* analysis (Chap. 4)
3. Experimental study, or *dimensional* analysis (Chap. 5)

In all cases, the flow must satisfy the three basic laws of mechanics[8] plus a thermodynamic state relation and associated boundary conditions:

1. Conservation of mass (continuity)
2. Linear momentum (Newton's second law)
3. First law of thermodynamics (conservation of energy)
4. A state relation like $\rho = \rho(p, T)$
5. Appropriate boundary conditions at solid surfaces, interfaces, inlets, and exits

In integral and differential analyses, these five relations are modeled mathematically and solved by computational methods. In an experimental study, the fluid itself performs this task without the use of any mathematics. In other words, these laws are believed to be fundamental to physics, and no fluid flow is known to violate them.

Chapter 2

If the fluid is at rest or at constant velocity, $\nabla p = \rho \mathbf{g}$

$$p_{down} = p^{up} + \gamma |\Delta z|$$

Any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure.

$$F = p_a A + \gamma h_{CG} A = (p_a + \gamma h_{CG}) A = p_{CG} A$$

The horizontal component of force on a curved surface equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.

The vertical component of pressure force on a curved surface equals in magnitude and direction the weight of the entire column of fluid, both liquid and atmosphere, above the curved surface.

1. A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces.
2. A floating body displaces its own weight in the fluid in which it floats.

$$(F_B)_{LF} = \sum \rho_i g (\text{displaced volume})_i$$

Chapter 3

$$Q = \int_S (\mathbf{V} \cdot \mathbf{n}) \, dA = \int_S V_n \, dA$$

$$\dot{m} = \int_S \rho(\mathbf{V} \cdot \mathbf{n}) \, dA = \int_S \rho V_n \, dA$$

Constant density: $\dot{m} = \rho Q$

Conservation of Mass $$0 = \frac{d}{dt}\left(\int_{CV} \rho \, d\mathcal{V}\right) + \int_{CS} \rho(\mathbf{V_r} \cdot \mathbf{n}) \, dA$$

If the control volume has only a number of one-dimensional inlets and outlets, we can write

$$\int_{CV} \frac{\partial \rho}{\partial t} \, d\mathcal{V} + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

Incompressible Flow $$\sum_i (V_i A_i)_{out} = \sum_i (V_i A_i)_{in}$$

$$\sum Q_{out} = \sum Q_{in}$$

Chapter 3

Linear Momentum

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho \, d\mathcal{V}\right) + \int_{CS} \mathbf{V}\rho(\mathbf{V}\cdot\mathbf{n})\, dA$$

One-Dimensional Momentum Flux

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho \, d\mathcal{V}\right) + \sum(\dot{m}_i \mathbf{V}_i)_{out} - \sum(\dot{m}_i \mathbf{V}_i)_{in}$$

Net Pressure Force on a Closed Control Surface

$$\mathbf{F}_{press} = \int_{CS} (p - p_a)(-\mathbf{n})\, dA = \int_{CS} p_{gage}(-\mathbf{n})\, dA$$

Momentum-Flux Correction Factor

$$\rho\int u^2 dA = \beta \dot{m} V_{av} = \beta\rho A V_{av}^2$$

$$\beta = \frac{1}{A}\int \left(\frac{u}{V_{av}}\right)^2 dA$$

Laminar flow: $u = U_0\left(1 - \frac{r^2}{R^2}\right)$ $\beta = \frac{4}{3}$

Turbulent flow: $u \approx U_0\left(1 - \frac{r}{R}\right)^m$ $\frac{1}{9} \le m \le \frac{1}{5}$

$$\sum \mathbf{F} = \dot{m}(\beta_2\mathbf{V}_2 - \beta_1\mathbf{V}_1)$$

Chapter 3

Angular-Momentum

$$\sum \mathbf{M}_0 = \frac{\partial}{\partial t}\left[\int_{CV} (\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V}\right] + \int_{CS} (\mathbf{r} \times \mathbf{V})\rho(\mathbf{V} \cdot \mathbf{n}) \, dA$$

one-dimensional inlets and exits $\int_{CS} (\mathbf{r} \times \mathbf{V})\rho(\mathbf{V} \cdot \mathbf{n}) \, dA = \sum (\mathbf{r} \times \mathbf{V})_{out} \, \dot{m}_{out} - \sum (\mathbf{r} \times \mathbf{V})_{in} \, \dot{m}_{in}$

Energy Equation

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\left[\int_{CV} \left(\hat{u} + \tfrac{1}{2} V^2 + gz\right) \rho \, d\mathcal{V}\right] + \int_{CS} \left(\hat{h} + \tfrac{1}{2} V^2 + gz\right) \rho(\mathbf{V} \cdot \mathbf{n}) \, dA$$

One-Dimensional Energy-Flux Terms

$$\int_{CS} (\hat{h} + \tfrac{1}{2}V^2 + gz)\rho(\mathbf{V} \cdot \mathbf{n}) \, dA = \sum (\hat{h} + \tfrac{1}{2}V^2 + gz)_{out}\dot{m}_{out} - \sum (\hat{h} + \tfrac{1}{2}V^2 + gz)_{in}\dot{m}_{in}$$

The Steady-Flow Energy Equation

$$\frac{p_1}{\gamma} + \frac{\hat{u}_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\hat{u}_2}{g} + \frac{V_1^2}{2g} + z_2 - h_q + h_s + h_v$$

Friction Losses in Low-Speed Flow

$$\left(\frac{p}{\rho g} + \frac{\alpha}{2g} V^2 + z\right)_{in} = \left(\frac{p}{\rho g} + \frac{\alpha}{2g} V^2 + z\right)_{out} + h_{turbine} - h_{pump} + h_{friction}$$

Kinetic-Energy Correction Factor

$$\int_{port} (\tfrac{1}{2}V^2)\rho(\mathbf{V} \cdot \mathbf{n}) \, dA \equiv \alpha(\tfrac{1}{2}V_{av}^2)\dot{m}$$

where $V_{av} = \frac{1}{A}\int u \, dA$ for incompressible flow

$$\alpha = \frac{1}{A}\int \left(\frac{u}{V_{av}}\right)^3 dA$$

Laminar flow: $u = U_0\left[1 - \left(\frac{r}{R}\right)^2\right]$

from which $V_{av} = 0.5U_0$

and $\alpha = 2.0$

Chapter 3

Frictionless Flow: Bernoulli Equation

$$\int_1^2 \frac{\partial V}{\partial t}\,ds + \int_1^2 \frac{dp}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0$$

steady ($\partial V/\partial t = 0$) incompressible (constant-density) flow

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = \text{const}$$

Relation between the Bernoulli and Steady-Flow Energy Equations

The complete list of assumptions

1. *Steady flow*—a common assumption applicable to many flows.
2. *Incompressible flow*—acceptable if the flow Mach number is less than 0.3.
3. *Frictionless flow*—very restrictive, solid walls introduce friction effects.
4. *Flow along a single streamline*—different streamlines may have different "Bernoulli constants" $w_0 = p/\rho + V^2/2 + gz$, depending upon flow conditions.
5. *No shaft work between 1 and 2*—no pumps or turbines on the streamline.
6. *No heat transfer between 1 and 2*—either added or removed.

Thus our warning: Be wary of misuse of the Bernoulli equation. Only a certain limited set of flows satisfies all six assumptions above. The usual momentum or "mechanical force" derivation of the Bernoulli equation does not even reveal items 5 and 6, which are thermodynamic limitations. The basic reason for restrictions 5 and 6 is that heat transfer and work transfer, in real fluids, are married to frictional effects, which therefore invalidate our assumption of frictionless flow.

Chapter 4

$$\mathbf{V}(\mathbf{r}, t) = \mathbf{i}u(x, y, z, t) + \mathbf{j}v(x, y, z, t) + \mathbf{k}w(x, y, z, t)$$

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \underbrace{\frac{\partial \mathbf{V}}{\partial t}}_{\text{Local}} + \underbrace{\left(u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}\right)}_{\text{Convective}} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}$$

Mass Conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Cylindrical Polar Coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Steady Compressible Flow

Cartesian:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Cylindrical:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Incompressible Flow

Cartesian:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Cylindrical:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

Chapter 4

Linear Momentum

$$\rho \mathbf{g} - \nabla p + \nabla \cdot \tau_{ij} = \rho \frac{d\mathbf{V}}{dt}$$

where
$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$$

$$\rho g_x - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\rho g_y - \frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$\rho g_z - \frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

$$\sigma_{ij} = \begin{vmatrix} -p + \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & -p + \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & -p + \tau_{zz} \end{vmatrix}$$

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{bmatrix}$$

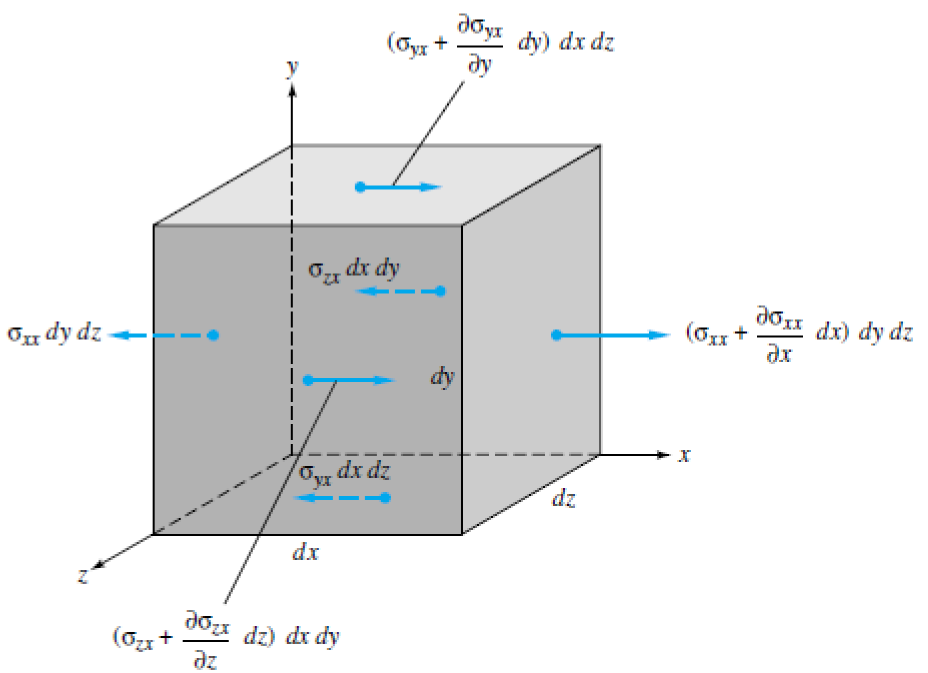


Fig. 4.4 Elemental cartesian fixed control volume showing the surface forces in the *x* direction only.

Chapter 4

Inviscid Flow: Euler's Equation $\rho \mathbf{g} - \nabla p = \rho \dfrac{d\mathbf{V}}{dt}$

Newtonian Fluid: Navier-Stokes Equations

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho \frac{du}{dt}$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho \frac{dv}{dt}$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho \frac{dw}{dt}$$

4.6 Boundary Conditions for the Basic Equations

Solid surface: $\mathbf{V} = \mathbf{V}_{wall}$

Inlet or outlet: Known $\mathbf{V}$, p

Free surface: $p \approx p_a \qquad w \approx \dfrac{\partial \eta}{\partial t}$

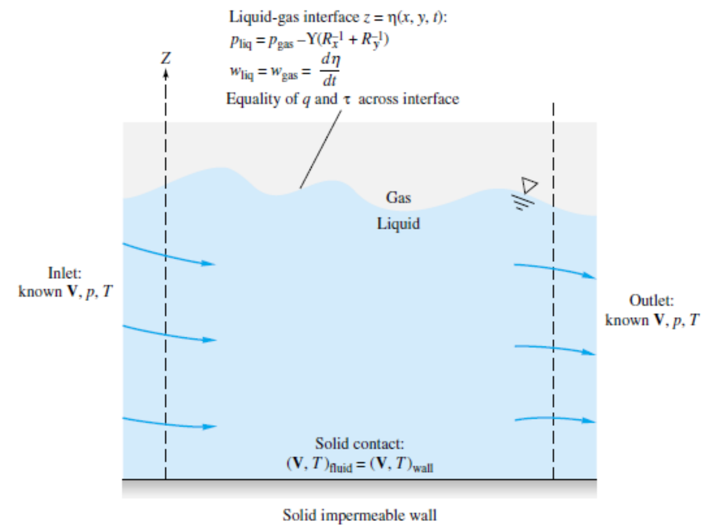


Fig. 4.7 Typical boundary conditions in a viscous heat-conducting fluid-flow analysis.

Chapter 5

The Pi Theorem

If a physical process satisfies the PDH and involves n dimensional variables, it can be reduced to a relation between only k dimensionless variables or Π's. The reduction $j = n - k$ equals the maximum number of variables which do not form a pi among themselves and is always less than or equal to the number of dimensions describing the variables.

Find the reduction j, then select j scaling variables which do not form a pi among themselves.[4] Each desired pi group will be a power product of these j variables plus one additional variable which is assigned any convenient nonzero exponent. Each pi group thus found is independent.

1. List and count the n variables involved in the problem. If any important variables are missing, dimensional analysis will fail.
2. List the dimensions of each variable according to $\{MLT\Theta\}$ or $\{FLT\Theta\}$. A list is given in Table 5.1.
3. Find j. Initially guess j equal to the number of different dimensions present, and look for j variables which do not form a pi product. If no luck, reduce j by 1 and look again. With practice, you will find j rapidly.
4. Select j scaling parameters which do not form a pi product. Make sure they please you and have some generality if possible, because they will then appear in every one of your pi groups. Pick density or velocity or length. Do not pick surface tension, e.g., or you will form six different independent Weber-number parameters and thoroughly annoy your colleagues.
5. Add one additional variable to your j repeating variables, and form a power product. Algebraically find the exponents which make the product dimensionless. Try to arrange for your output or *dependent* variables (force, pressure drop, torque, power) to appear in the numerator, and your plots will look better. Do this sequentially, adding one new variable each time, and you will find all $n - j = k$ desired pi products.
6. Write the final dimensionless function, and check your work to make sure all pi groups are dimensionless.