Name: ---------------- **Exam 1** **Time: 50 minutes**

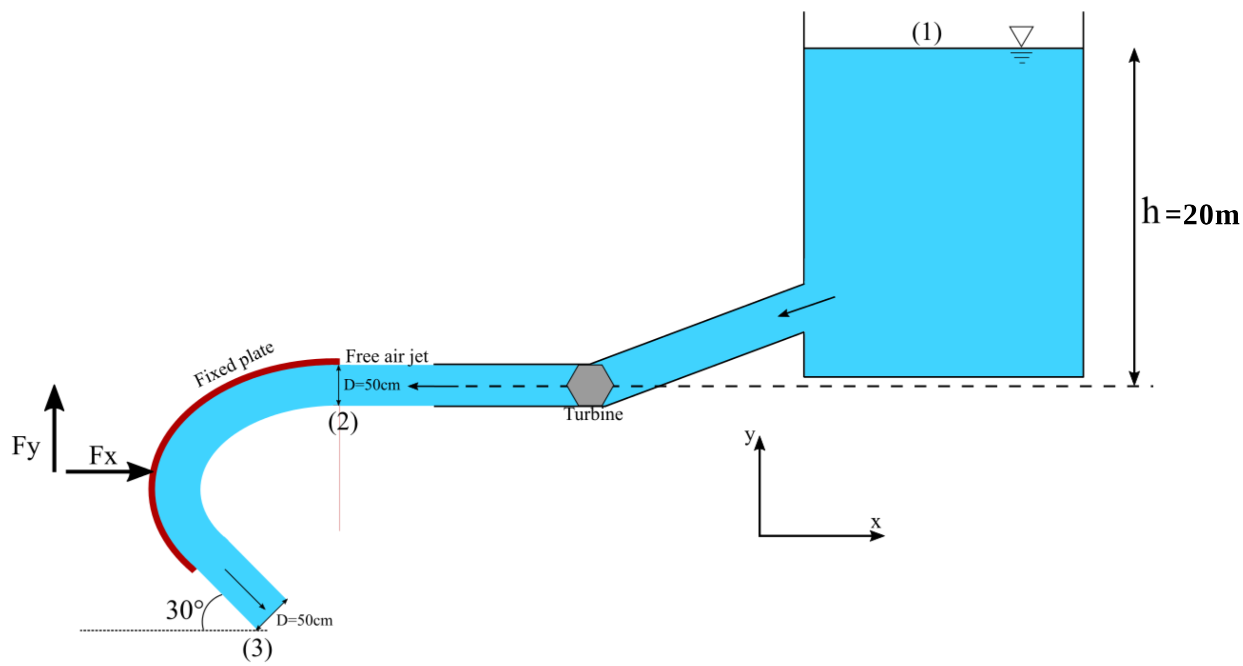
ME:5160 **Fall 2024**

The exam is closed book and closed notes.

1. A reservoir of water (ρ = 998 kg/m^3) discharges through a conduct (D=50cm), where a turbine is placed to generate hydroelectric energy. System friction losses between section (1) and (2) are $h_f = K{V_2}^2/(2g)$. Assume $\alpha = 1$ for both sections. (a) Find an expression for h_t as a function of V_2. (b) Suppose that $h_t = 17.9m$ and $K = 3.5$, what is the correspondent value of V_2?
The exhaust water from the turbine is guided using a fixed plate, as shown in the Figure. (c) Determine the horizontal and vertical component of the force that the water jet exerts on the plate.

Hint:

Energy equation: $\left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_1 = \left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_2 + h_f + h_t$



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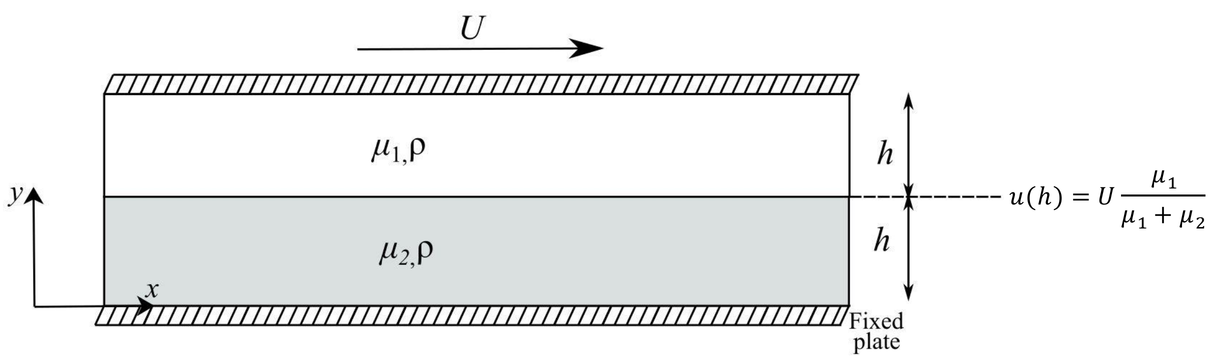
2. Two immiscible, incompressible, viscous fluids having the same densities, but different viscosities are contained between two infinite, horizontal, parallel plates, as shown in the Figure. Pressure gradient is negligible, and the flow is fully developed ($\frac{\partial}{\partial x} = 0$). The bottom plate is fixed and the upper plate moves with a constant velocity U.

The velocity at the interface is equal to $u(h) = U\frac{\mu_1}{\mu_1+\mu_2}$ for both fluids.

Determine (a) the velocity field in both fluids and (b) evaluate the shear stress in both fluids at y=h. (c) What can you notice for the shear stress value?

Hint: Solve NS for each fluid, separately, using appropriate BCs for the velocity at the upper, lower wall and fluid interface.



Incompressible continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Incompressible Navier-Stokes Equations in Cartesian Coordinates:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(u\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\left(v\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\left(w\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

Shear stress: $\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

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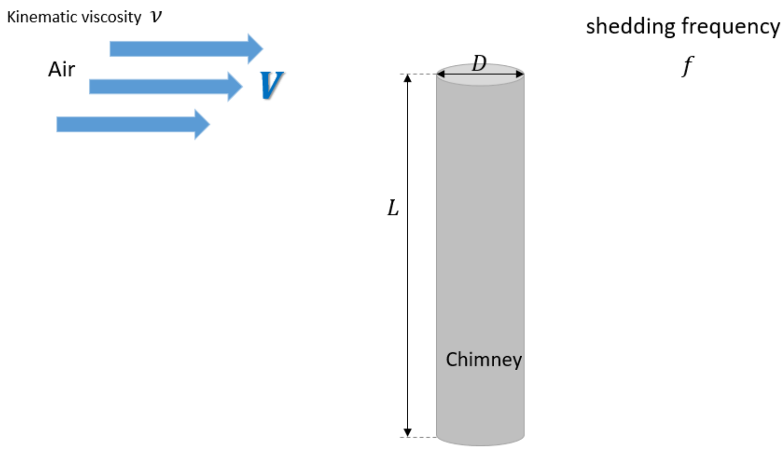
3. As wind blows over a chimney, vortices are shedding in the wake as shown in the Figure below. The dimensional shedding frequency f depends on chimney diameter D, chimney length L, wind velocity V, and air kinematic viscosity ν. (a) Find dimensionless f which depends on dimensionless groups.



Quantity	Symbol	Dimensions $MLT\Theta$	Dimensions $FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

Solution 1:

1. (a) Apply energy equation between (1) and (2):

$$\left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_2+h_f+h_t \quad \textbf{(+2)}$$

$$(0+0+z)_1=\left(0+\frac{V^2}{2g}+z\right)_2+h_f+h_t$$

$$h_t=h-\frac{{V_2}^2}{2g}-K\frac{{V_2}^2}{2g}=h-\frac{{V_2}^2}{2g}(1+K)$$

(+2)

(b) Substitute K=3.5, $h_t=17.9m \rightarrow V_2=3m/s.$ **(+1)**

(c) Using continuity: $A_2V_2=A_3V_3$, but $D_2=D_2 \rightarrow V_2=V_3$ **(+1)**

$$\sum \boldsymbol{F}=\frac{d}{dt}\left(\int_{CV}\boldsymbol{V}\rho dV\right)+\int_{CS}\boldsymbol{V}\rho(\boldsymbol{V_r}\cdot\boldsymbol{n})dA$$

Assuming fixed control volume, $\boldsymbol{V_r}=\boldsymbol{V}$**,** constant density and zero acceleration

$$\sum \boldsymbol{F}=\int_{CS}\boldsymbol{V}\rho(\boldsymbol{V}\cdot\boldsymbol{n})dA$$

Assuming V and ρ uniform over $A_2$and A_3 and projecting in the x-direction:

$$\sum F_x=-F_x=\int_{CS}u\rho(\boldsymbol{V}\cdot\boldsymbol{n})dA=-V_2\rho(\boldsymbol{V_2}\cdot\boldsymbol{n_2})A_2+V_3\cos(30°)\,\rho(\boldsymbol{V_3}\cdot\boldsymbol{n_3})A_3$$

$$-F_x=\rho{V_2}^2A_2+\rho{V_3}^2\cos(30°)\,A_3=\rho{V_2}^2A_2(1+\cos(30°)) \quad \textbf{(+1)}$$

$$F_x=-\frac{998\cdot 3^2\cdot\pi\cdot(0.5m)^2}{4}(1.866)=-3289N \quad \textbf{(+1)}$$

Projecting in the y-direction:

$$\sum F_y=-F_y=\int_{CS}u\rho(\boldsymbol{V}\cdot\boldsymbol{n})dA=-V_3\sin(30°)\,\rho(\boldsymbol{V_3}\cdot\boldsymbol{n_3})A_3$$

$$-F_y=-\rho{V_3}^2\sin(30°)\,A_3 \quad \textbf{(+1)}$$

$$F_y=\frac{998\cdot 3^2\cdot\pi\cdot(0.5m)^2}{4}(0.5)=881N \quad \textbf{(+1)}$$

Solution 2:

Assumptions:

1. Steady flow ($\frac{\partial}{\partial t} = 0$)
2. 2D flow (w=0)
3. Incompressible flow (ρ =constant)
4. Fully developed flow ($\frac{\partial u}{\partial x} = 0$)
5. Pressure gradient is negligible (∇P=0)

(a) x-momentum for fluid 2:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2}\right) = \rho\left(u_2\frac{\partial u_2}{\partial t} + u_2\frac{\partial u_2}{\partial x} + v_2\frac{\partial u_2}{\partial y}\right) \quad \textbf{(+1)}$$

$$\frac{\partial^2 u_2}{\partial y^2} = 0 \quad \textbf{(+1.5)}$$

$$u_2(y) = Ay + B \quad \textbf{(+1)}$$

Apply BCs:

$$u_2(0) = 0 \quad \textbf{(+0.5)}$$

$$u_2(h) = U\frac{\mu_1}{\mu_1 + \mu_2} \quad \textbf{(+0.5)}$$

$$A = \frac{U}{h}\frac{\mu_1}{\mu_1 + \mu_2} \quad \textbf{(+0.5)}$$

$$B = 0 \quad \textbf{(+0.5)}$$

$$u_2(y) = \frac{U}{h}\frac{\mu_1}{\mu_1 + \mu_2}y \quad \textbf{(+0.5)}$$

x-momentum for fluid 1:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}\right) = \rho\left(u_1\frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y}\right)$$

$$\frac{\partial^2 u_1}{\partial y^2} = 0$$

$$u_1(y) = Cy + D$$

Apply BCs:

$$u_1(h) = U\frac{\mu_1}{\mu_1 + \mu_2}$$ **(+0.5)**

$$u_1(2h) = U$$ **(+0.5)**

$$C = \frac{U}{h}\frac{\mu_2}{\mu_1 + \mu_2}$$ **(+0.5)**

$$D = U\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$$ **(+0.5)**

$$u_1(y) = \frac{U}{h}\frac{\mu_2}{\mu_1 + \mu_2}y + U\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$$ **(+0.5)**

(b) Evaluate shear stress at y=h:

$$\tau_1(h) = \mu_1\frac{\partial u_1}{\partial y} = \mu_1\frac{U}{h}\frac{\mu_2}{\mu_1 + \mu_2}$$ **(+0.5)**

$$\tau_2(h) = \mu_2\frac{\partial u_2}{\partial y} = \mu_2\frac{U}{h}\frac{\mu_1}{\mu_1 + \mu_2}$$ **(+0.5)**

The shear stress value is the same. Therefore, shear stress is continuous across the two fluids. **(+0.5)**

Solution 3:

ASSUMPTIONS: the problem is only a function of the above dimensional variables

ANALYSIS:
(a)

$$f = fcn(D, L, V, \nu) \ ; \quad n = 5$$

$$f = \{T^{-1}\} \quad D = \{L\} \quad L = \{L\} \quad V = \{LT^{-1}\} \quad \nu = \{L^2 T^{-1}\} \ ; \quad j = 2$$

$$\therefore k = n - j = 3$$ **(2.5)**

$$repeating\ variables = D, V$$

$$\pi_1 = f D^{a_1} V^{b_1} = \{(T^{-1})(L)^{a_1}(LT^{-1})^{b_1}\} = \{L^0 T^0\}$$

$$a_1 = 1; \ \ b_1 = -1$$

$$\pi_1 = \frac{fD}{V}$$ **(2.5)**

$$\pi_2 = L D^{a_2} V^{b_2} = \{(L)(L)^{a_2}(LT^{-1})^{b_2}\} = \{L^0 T^0\}$$

$$a_2 = -1; \ \ b_2 = 0$$

$$\pi_2 = \frac{L}{D}$$ **(2.5)**

$$\pi_3 = \nu D^{a_3} V^{b_3} = \{(L^2 T^{-1})(L)^{a_3}(LT^{-1})^{b_3}\} = \{L^0 T^0\}$$

$$a_3 = -1; \ \ b_3 = -1$$

$$\pi_3 = \frac{\nu}{VD}$$ **(2.5)**

$$\frac{fD}{V} = fcn\left(\frac{L}{D}, \frac{\nu}{VD}\right)$$