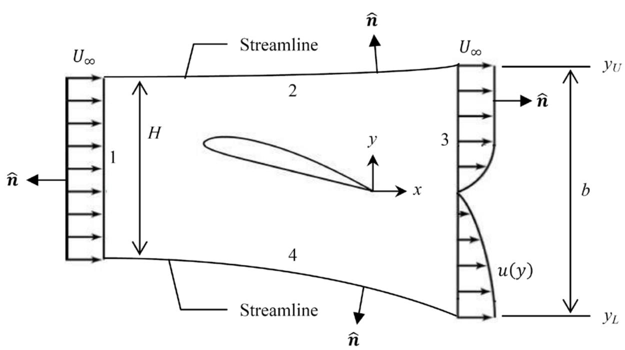
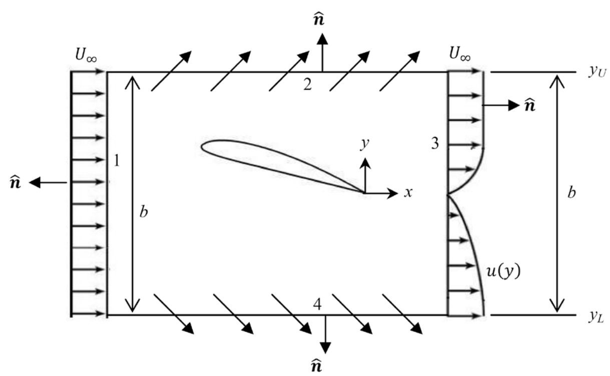
Experimental Determination Profie Drag by Simplified Betz Method

The drag force on an airfoil is measured in a wind tunnel using both a load cell and a simplified Betz method and the results are compared. Steady incompressible constant property conditions are assumed. The drag force can be determined from the measurement of the uniform velocity upstream and nonuniform velocity distribution downstream of the airfoil.

1. The velocity far upstream is the uniform flow U
2. The nonuniform velocity distribution downstream of the airfoil is measured using hotwire and/or Pitot probe is $u(y)$, which is less than U in the wake region due to the drag of the airfoil.
3. Uniform atmospheric pressure is assumed on all the control volumes boundaries
4. Objective: Find the drag force D per unit span length of the airfoil using the simplified Betz method.



(a) Streamline CV



(b) Box CV

Figure 1 Continuity and Momentum Equation Control Volumes

First, we show that either streamline of box control volumes can be used; however, the box control volume is preferred. Second, example results are shown from previous http://user.engineering.uiowa.edu/~fluids/ EFD Lab3. The elaborated Betz method accounts for a upstream nonuniform velocity distribution and both upstream and downstream nonuniform pressure distributions.

① Ψ CV

$$\int_{CS} \rho \underline{V}\cdot\underline{n}\, dA = 0 = -\int_{-H/2}^{H/2} \rho U\, dy + \int_{y_l}^{y_u} u\, dy$$

$$UH = \int_{y_l}^{y_u} u\, dy$$

$$-D = \int_{CS} \rho u \underline{V}\cdot\underline{n}\, dA = -\int_{-H/2}^{H/2} \rho U (U\, dy) + \int_{y_l}^{y_u} \rho u (u\, dy)$$

$$D = \rho U^2 H - \int_{y_l}^{y_u} \rho u^2\, dy$$

$$= \rho U^2 \int_{y_l}^{y_u} \frac{u}{U} dy - \int_{y_l}^{y_u} \rho u^2 dy = \rho U^2 \int_{y_l}^{y_u} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \rho \int_{y_l}^{y_u} u (U - u)\, dy$$

② Box CV

$$0 = -\int_1 \rho U dy + \int_3 \rho u dy + \underbrace{\int_2 \rho v dx}_{\dot m_t} + \underbrace{\int_4 \rho(-v dx)}_{\dot m_b}$$

$$\dot m_t + \dot m_b = \rho U S - \int_3 \rho u dy$$

assume $u = U$: $= U(\dot m_t + \dot m_b)$

$$-D = -\int_1 \rho U (U dy) + \int_3 \rho u (u dy) + \int_2 \rho u (v dx) + \int_4 \rho u (-v dx)$$

$$D = \cancel{\rho U^2 S} - \int_3 \rho u^2 dy - U\left(\cancel{\rho U S} - \int_3 \rho u dy\right)$$

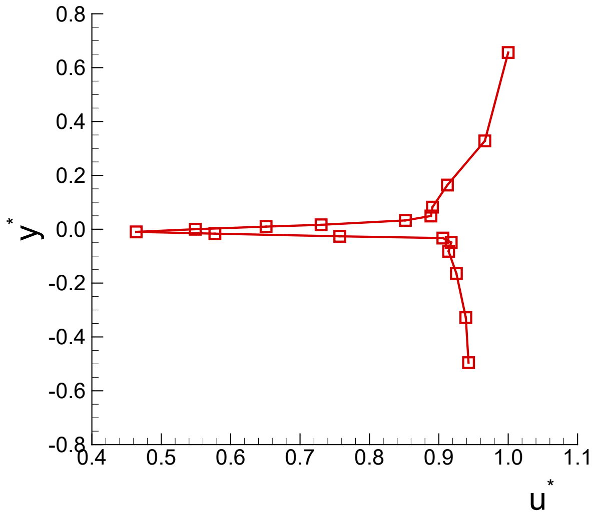
$$= U \int_3 \rho u dy - \int_3 \rho u^2 dy = \rho \int_{y_l}^{y_u} u (U - u)\, dy \quad \text{per unit span}$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 c s} = 2 s \int_{y_l}^{y_u} u(1-u)\, dy \qquad \div s \text{ for per unit span}$$

c = cord length, s = span length, $y = y/c$, $u = u/U$

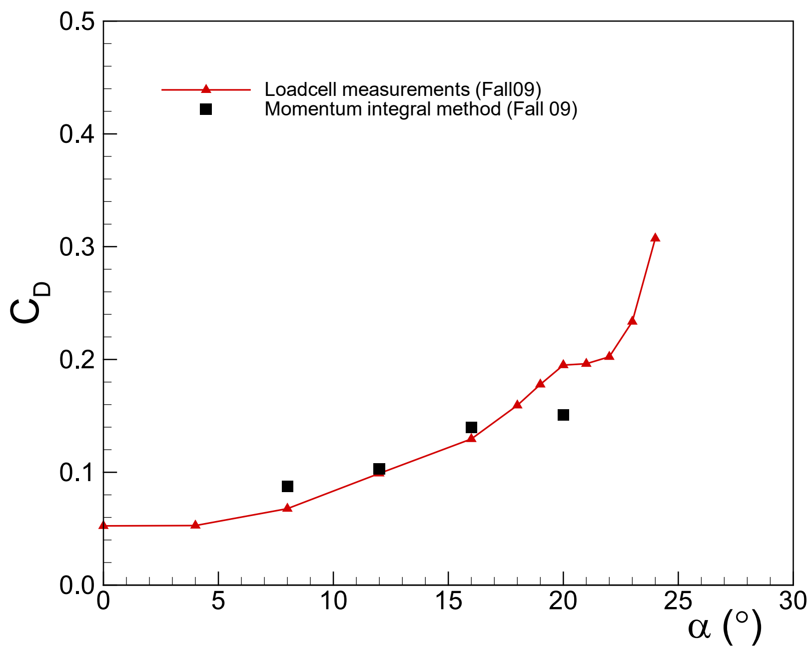
Example

U_∞ = 14.4 m/s, c = 0.3048 m, AOA = 16°



i	y_i (m)	u_i (m/s)	y_i^*	u_i^*
1	0.200	14.4384	0.65617	1.00000
2	0.100	13.9520	0.32808	0.96631
3	0.050	13.1723	0.16404	0.91231
4	0.025	12.8620	0.08202	0.89082
5	0.015	12.8298	0.04921	0.88859
6	0.010	12.2982	0.03281	0.85178
7	0.005	10.5453	0.01640	0.73037
8	0.003	9.4002	0.00984	0.65106
9	0.000	7.9273	0.00000	0.54904
10	-0.003	6.6970	-0.00984	0.46383
11	-0.005	8.3346	-0.01640	0.57725
12	-0.008	10.9333	-0.02625	0.75724
13	-0.010	13.0791	-0.03281	0.90586
14	-0.015	13.2519	-0.04921	0.91783
15	-0.025	13.1977	-0.08202	0.91407
16	-0.050	13.3596	-0.16404	0.92529
17	-0.100	13.5565	-0.32808	0.93892
18	-0.151	13.6128	-0.49541	0.94282

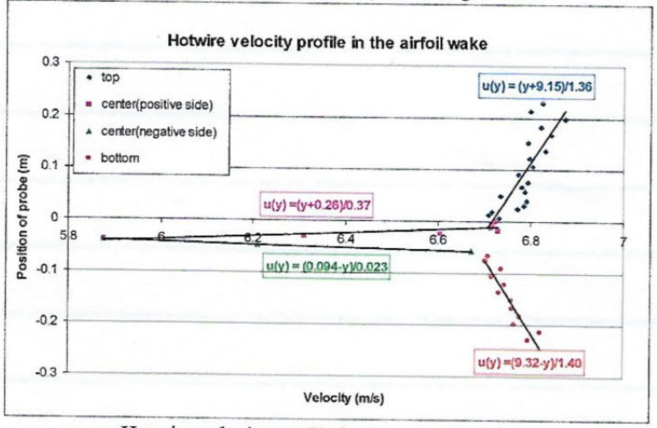
Pitot measured velocity profile (left) and the measurement data (right), where $y^* = 0$ is at the trailing edge (TE) of the wing model, and the measurement is at about one inch behind the TE.



Comparisons of the drag coefficient C_D

Example

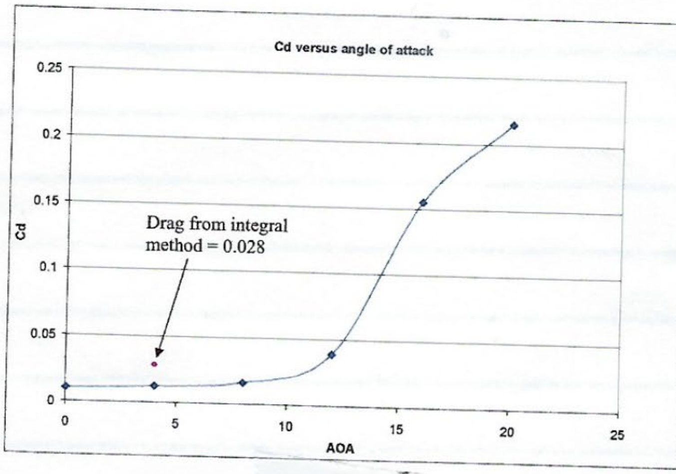
$U_\infty = 7.04$m/s, b = 0.762m, c = 0.3048m, $\rho = 1.21$ kg/m^3



Hotwire velocity profile in the wake for AOA = 4

$$C_D = \frac{4\int_0^b u(U_\infty - u)dy}{U_\infty^2 bc} = 2\times 0.174\left[\begin{array}{l}\left(\int_{0.006}^{0.229}(0.735y+6.73)(0.31-0.735y)dy\right)+\\ \left(\int_{-0.039}^{0.0}(2.7y+0.702)(6.34-02.7y)dy\right)+\\ \left(\int_{-0.058}^{-0.039}(4.09y-4.48)(5.52-4.09y)dy\right)+\\ \left(\int_{-0.229}^{-0.067}(6.66y-0.714)(7.75-6.66y)dy\right)\end{array}\right]$$

*Note: The velocity profile in the wake is **not symmetrical** due to airfoil shape and angle of attack. Each of the four equations has different **y limits**.*



Comparison of drag data with benchmark