1.49 An amazing number of commercial and laboratory devices have been developed to measure fluid viscosity, as described in Ref. 27. Consider a concentric shaft, as in Prob. 1.47, but now fixed axially and rotated inside the sleeve. Let the inner and outer cylinders have radii r_i and r_o, respectively, with total sleeve length L. Let the rotational rate be Ω (rad/s) and the applied torque be M. Using these parameters, derive a theoretical relation for the viscosity μ of the fluid between the cylinders.

Solution: Assuming a linear velocity distribution in the annular clearance, the shear stress is

$$\tau = \mu\frac{\Delta V}{\Delta r} \approx \mu\frac{\Omega r_i}{r_o - r_i}$$

This stress causes a force $dF = \tau\, dA = \tau(r_i\, d\theta)L$ on each element of surface area of the inner shaft. The moment of this force about the shaft axis is $dM = r_i\, dF$. Put all this together:

$$M = \int r_i\, dF = \int_0^{2\pi} r_i \mu \frac{\Omega r_i}{r_o - r_i} r_i\, L\, d\theta = \frac{2\pi\mu\Omega r_i^3 L}{r_o - r_i}$$

Solve for the viscosity: $\boldsymbol{\mu \approx M(r_o - r_i)/\{2\pi\Omega r_i^3 L\}}$ *Ans.*