Chapter 8: Channel and Pipe Flows (Chapter 7.1-7.2 Pope)

Part 2: Pipe Flow

In polar cylindrical coordinates (x, r, θ) for fully developed turbulent flow, velocity statistics depend solely on r.

The mean centerline velocity is denoted by U_0:

$$U_0 = \langle U(x, 0, \theta) \rangle$$

and the bulk velocity is:

$$\overline{U} = \frac{1}{\pi R^2} \int_0^R \langle U \rangle 2\pi r dr$$

Reynolds number:

$$Re = \frac{\overline{U} D}{\nu} = \frac{2\overline{U}\delta}{\nu}$$

Define

$$y = R - r$$

$y/\delta < 0.1\ u^+ = f_w(y^+)$

$y^+ > 30 \quad u^+ = k^{-1} \log y + B$, profiles follow log law for a range of y^+ that increases with Re and above log law near pipe centerline.

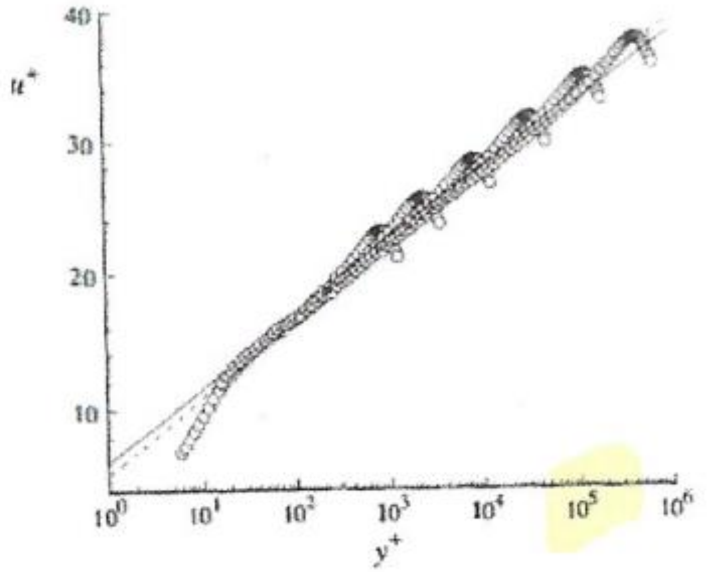


Fig. 7.20. Mean velocity profiles in fully developed turbulent pipe flow. Symbols, experimental data of Zagarola and Smits (1997) at six Reynolds numbers (Re $\approx 32 \times 10^3$, 99×10^3, 409×10^3, 1.79×10^6, 7.71×10^6, 29.9×10^6). Solid line, log law with $\kappa = 0.436$ and $B = 6.13$; dashed line, log law with $\kappa = 0.41$, $B = 5.2$.

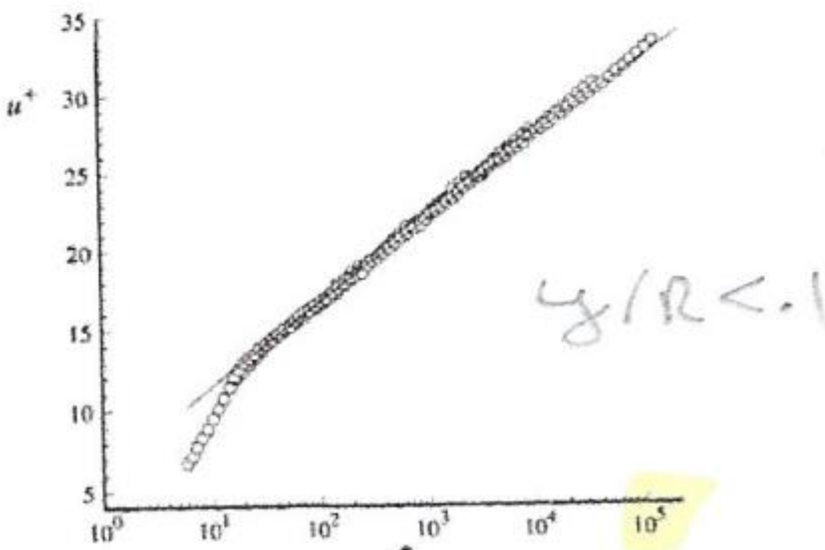


Fig. 7.21. Mean velocity profiles in fully developed turbulent pipe flow. Symbols, experimental data of Zagarola and Smits (1997) for $y/R < 0.1$, for the same values of Re as in Fig. 7.20. Line, log law with $\kappa = 0.436$ and $B = 6.13$.

Friction factor f:

$$f = \frac{\Delta p D}{\frac{1}{2}\rho \overline{U}^2 L}$$

$$\Delta h = \Delta\left(\frac{P}{\gamma} + z\right) = f\frac{L}{D}\frac{V^2}{g} \quad \text{Darcy-Weisbach equation}$$

Prandtl's friction law for smooth pipes:

$$\frac{1}{\sqrt{f}} = 2.0\log_{10}(\sqrt{f}\,Re) - 0.8$$

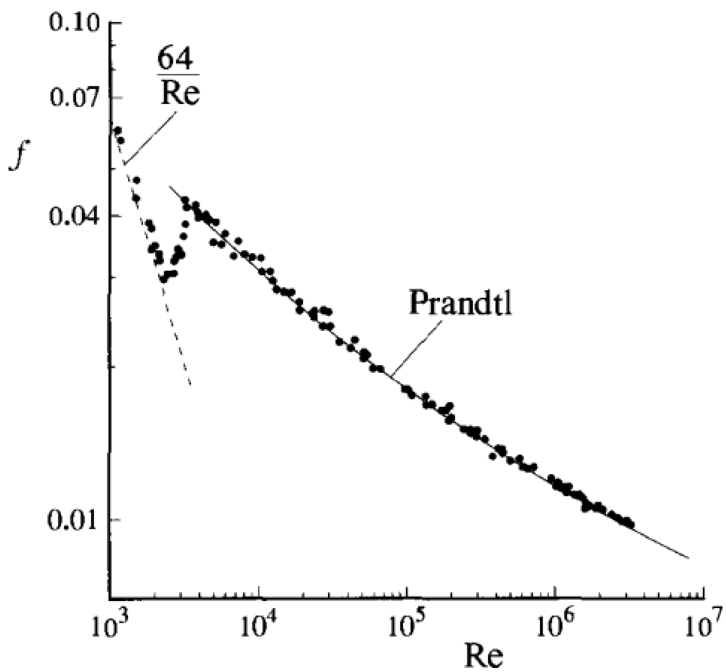


Fig. 7.22. The friction factor f against the Reynolds number for fully developed flow in smooth pipes. Dashed line, Hagen–Poiseuille friction law for laminar flow; solid line, Prandtl friction law for turbulent flow, Eq. (7.98); symbols, measurements compiled by Schlichting (1979). (Reproduced with permission of McGraw-Hill.)

Wall roughness

s/R no effect laminar or transition

Smooth pipe up to certain Re after which turn upward and reach asymptotes.

At higher Re, f is independent of Re.

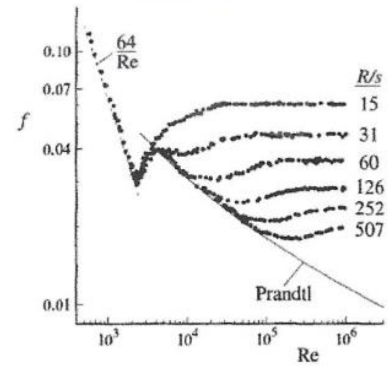


Fig. 7.23. The friction factor f against the Reynolds number for fully developed flow in pipes of various roughnesses. Dashed line, friction law for laminar flow; solid line, Prandtl friction law for turbulent flow in smooth pipes, Eq. (7.98); symbols, measurements of Nikuradse. (Adapted from Schlichting (1979) with permission of McGraw-Hill.)

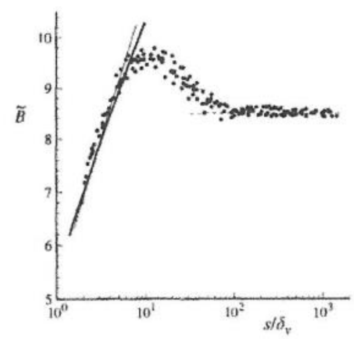


Fig. 7.24. The additive constant in the log law $\tilde{B}$ (Eq. (7.121)) as a function of the roughness scale s normalized by the viscous length δ_ν. Dashed line, fully rough $\tilde{B} = 8.5$; solid line, smooth (Eq. (7.122)); symbols, from Nikuradse's data. (Adapted from Schlichting (1979) with permission of McGraw-Hill.)

This can be explained using extended log law:

$$\langle U \rangle_y = \frac{u_\tau}{y} \Phi\left(\frac{y}{\delta_\nu}, \frac{y}{\delta}, \frac{s}{\delta_\nu}\right) \qquad \boxed{\delta = R}$$

$\Phi =$ universal non dimensional function $\neq f(y/\delta)$ for $(y/\delta) < 0.1$

At high Re, two extreme cases can be considered:

1) $\frac{s}{\delta_\nu} \ll 1 \rightarrow$ no effect roughness

$$\langle U \rangle_y = \frac{u_\tau}{y} \Phi\left(\frac{y}{\delta_\nu}\right) \quad s \ll \delta_\nu \text{ and } y \ll \delta$$

For large $\frac{y}{\delta_\nu} \rightarrow\ \Phi = \text{constant} \sim 1/k$ and log law is recovered:

$$\langle U \rangle_y = \frac{u_\tau}{ky}$$

$$\frac{\langle U \rangle_y}{u_\tau} = \frac{du^+}{dy^+}\frac{dy^+}{dy} = \frac{1}{ky}$$

$$u^+ = \frac{1}{k}\log y^+ + B \quad s \ll \delta_\nu \ll y \ll \delta$$

Where:

$$B = \lim_{y^* \to \infty}\left\{\int_0^{y^*} \Phi_I(y^+)\frac{dy^+}{y^+} - \frac{1}{k}\log y^+\right\}$$

2) $\frac{s}{\delta_\nu} \gg 1$ pressure drag due to s causes an increase in f.

$$\langle U \rangle_y = \frac{u_\tau}{y}\Phi_R\left(\frac{y}{s}\right) \quad \delta_\nu \ll s \text{ and } y \ll \delta$$

For $y \gg s \rightarrow \Phi_R \neq f(s)$:

$$u^+ = \frac{1}{k}\log\left(\frac{y}{\delta}\right) + B_2 \quad \delta_\nu \ll s \ll y \ll \delta$$

where:

$$B_2 = \lim_{y^* \to \infty}\left\{\int_0^{y^*} \Phi_R\left(\frac{y}{s}\right)\frac{dy}{y} - \frac{1}{k}\log\left(\frac{y}{s}\right)\right\}$$

A third case is given by $s \sim \delta_\nu$:

$$u^+ = \frac{1}{k}\log\left(\frac{y}{s}\right) + \tilde{B}\left(\frac{s}{\delta_\nu}\right)$$

For smooth wall ($s/\delta_\nu \ll 1$):

$$\tilde{B}\left(\frac{s}{\delta_\nu}\right) = B + \frac{1}{k}\log\left(\frac{y}{s}\right)$$

For fully rough wall ($s/\delta_\nu \gg 1$):

$$\tilde{B}\left(\frac{s}{\delta_\nu}\right) = B_2$$

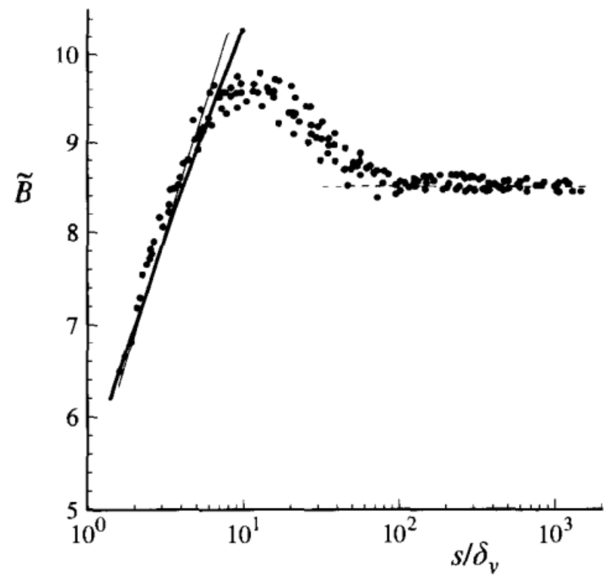


Fig. 7.24. The additive constant in the log law $\tilde{B}$ (Eq. (7.121)) as a function of the roughness scale s normalized by the viscous length δ_ν. Dashed line, fully rough $\tilde{B} = 8.5$; solid line, smooth (Eq. (7.122)); symbols, from Nikuradse's data. (Adapted from Schlichting (1979) with permission of McGraw-Hill.)

$s/\delta_\nu > 70$ wall fully rough $\rightarrow B_2 = \tilde{B}(\infty) = 8.5$

$s/\delta_\nu < 5$ smooth $\rightarrow \tilde{B}\left(\frac{s}{\delta_\nu}\right) = B + \frac{1}{k}\log\left(\frac{y}{s}\right)$

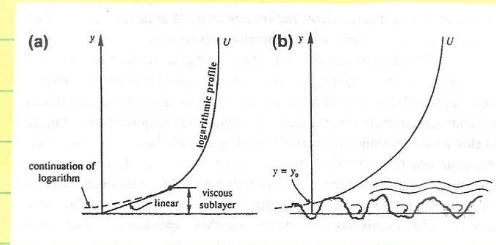
Fully rough:

$$f = \frac{1}{[1.99\log_{10}(R/s) + 1.71]^{2}}$$

Roughness

k = roughness height

$k^+ = \frac{k u^*}{\nu}$

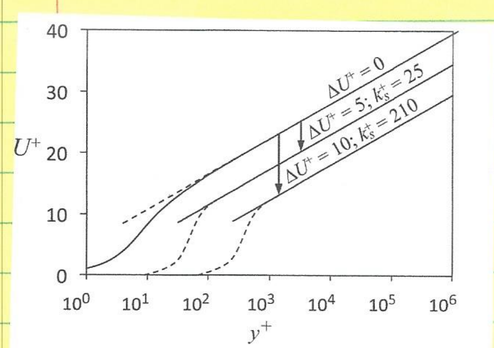


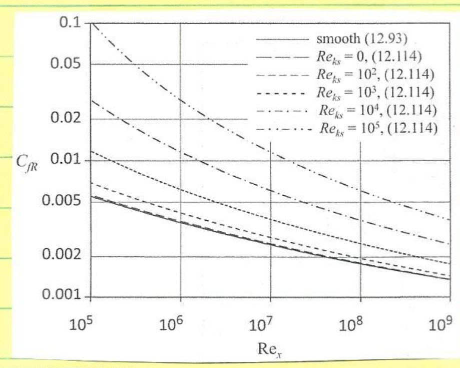
$k^+ < 4$ hydraulically smooth

$4 < k^+ < 60$ transitional roughness $f(Re)$

$k^+ > 0$ fully rough $\neq f(Re)$

$$U/u^* = \frac{1}{\kappa} \ln y^+ + B + \frac{2\Pi}{\kappa} W(y/\delta) - \Delta U^+$$





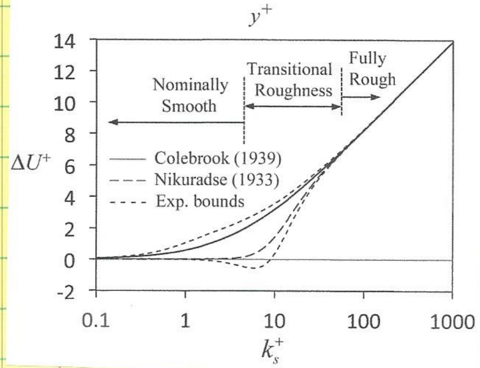


FIGURE 12.25 Rough surface skin friction coefficient, C_{fR}, for a zero-pressure-gradient flat-plate turbulent boundary layer vs. Re_x, the Reynolds number based on downstream distance. The solid curve corresponds to (12.93) evaluated using log-law constants $\kappa = 0.39$ and $B = 4.3$ (as recommended by Marusic et al., 2013). The dashed and dash-dot curves come from implicit evaluation of (12.114) for equivalent-sand-grain roughness-height Reynolds numbers of $Re_{ks} = 0, 10^2, 10^3, 10^4$, and 10^5. The C_{fR} values produced by (12.114) agree within engineering accuracy (± 5% or so) with prior rough-plate results.